

# MODELING THE VOLATILITY IN THE CENTRAL BANK RESERVES

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## ABSTRACT

The main idea of the research is to model the volatility of the Central Bank reserves. An important implication of modeling the volatility in an emerging market's Central Bank reserves is firstly, sharp volatility changes are seen in the reserves, which is not common in developed markets and secondly, to see the sensitivity of variables to the magnitude and the sign of the reserves. Measuring the size of the volatility and observing its determinants is important in determining the guidelines for monetary policy. For a Central Bank that has a precautionary motive it is important to monitor the increase in the volatility. In this respect, policy maker can determine the level of reserves in advance and take the necessary measures to stabilize the volatility in the financial markets. The effects of the volatility in the stock market and foreign currency markets are examined where overnight interest rates are introduced in the models as exogenous policy variable of the monetary policy and considered as the opportunity cost variable. It is found that overnight rates and the volatility in the exchange rate have negative effects on reserves. It is interesting to see that volatility at the Istanbul Stock Exchange (ISE) does not affect reserves at all; however, the return (percentage change in the stock market index) has a significant effect. On the other hand, out-of-sample forecasts present satisfactory results that indicated that the model chosen has a good forecasting performance.

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**Keywords:** reserves, ARCH, ISE-CI, exchange rate, overnight interest rates, volatility, monetary policy, forecasting.

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## **I. INTRODUCTION**

The main objective of the Central Bank is to maintain the stability in financial markets. For countries that are aiming at price stability through exchange rate adjustments, it is important to monitor foreign exchange markets and pursue an exchange rate policy that would be in line with monetary policy goals. An important instrument of the Central Bank to intervene in foreign exchange markets is the use of the Central Bank reserves.

Emerging open economies inviting high short-term capital flows are more vulnerable to external shocks. Altering expectations result in rapid inflows and outflows of capital. This results in an increase in the volatility of foreign exchange markets and eventually immediate reflection in the increasing volatility in the Central Bank reserves. This is due to the Central Bank's frequent intervention in the markets to adjust the exchange rate.

The volatility and trend in other markets such as the stock market is expected to affect reserves. If volatility increases in the stock market due to a capital outflow from the country, this should immediately be reflected in other markets. In addition, general confidence in other markets is reflected in the level of reserves.

Central Bank has basically two tools to smooth out the increasing volatility; the first one is the purchase and sale of foreign exchange, and the other is the rise and fall in the interest rates. It is expected that the relationship between the interest rate and reserves is negative. Firstly, because of the

ineffectiveness of the rise in the interest rate to halt capital outflows during the periods of crises and secondly, it represents the alternative cost of holding reserves.

An important implication of modeling the volatility in an emerging market's Central Bank reserves is firstly, sharp volatility changes are seen in the reserves, which is not common in developed markets and secondly, to see the sensitivity of variables to the magnitude and the sign of the reserves which will determine the guidelines for monetary policy. In the forecasting era, the policy maker will have foresight in determining the level of reserves and take the necessary actions to be inline with the monetary policy goals.

We were able to successfully model the dynamics of international reserves using the GARCH framework. This model improves our forecasting ability in the short-run. Findings of this research show that growth rate of reserves is negatively affected by the exchange rate volatility as expected. However, we were not able to find any significant relationship between the growth rate of reserves and the volatility in the stock market. Rather, the confidence in the stock market which is reflected by return is significantly related to reserves. The overnight interest rate, introduced as the policy variable of the Central Bank, is negatively related to the growth rate of reserves.

The volatility in the reserves is considered within ARCH and its variations framework. The policy variable of the Central Bank is considered as the overnight rates. Moreover, the volatility and return in the stock market and the volatility in the

exchange rate market are introduced in the model to see their possible effects.

The main motivation of the paper lies on the assumption that increasing volatility in foreign currency markets decreases stability in other markets. In this respect, Central Bank actively intervenes in these markets to stabilize. So that measuring the size of the volatility and observing the determinants is important in the conduct of the monetary policy.

The point is also mentioned in Edwards (1983) where main motive to hold international reserves for a Central Bank is to finance international transactions and correct unexpected difficulties in payments. The volatility in the external payments which puts pressure on the exchange rate, and the degree of openness of a country determine the level of reserves. On the other hand, since holding reserves implies forgone income, the higher the opportunity cost of reserves the lower the reserves be. In most cases the domestic interest rate is taken to be the cost of reserves.

Studies regarding the reserves of the Central Bank mainly focus on determining the optimal size of the reserves through using reserve demand equations. They stressed the importance of reserve holdings especially for precautionary motives. Special emphasis are given to developing countries, since they face payment imbalances more frequently than the developed countries. The models derive the specifications by including variables such as the interest rates, imports over GNP ratio and a measure of balance of payment variability

(Edwards, 1983), (Levy, 1983) and (Ben-Bassat and Gottlieb, 1992). The main finding of these studies is the negative relationship between interest rates and the level of reserves, since the interest rate represents the alternative cost of holding reserves, a positive relationship with the measure of variability. However, the sign of import could not be determined. This is mainly due to the level of imports showing the degree of openness of the country. This results in vulnerability of the country to external shocks where the sign come out to be positive (Frenkel, 1974). However, if imports represent the aggregate expenditure reacting to correcting payment imbalances, its sign should be negative (Heller, 1966).

Ben-Bassat and Gottlieb (1992) derived an optimal demand for precautionary reserves for a borrowing country that helped to explain the cost of default and country risk. They have employed a methodology different from previous work. They believe that reserve depletion has higher costs to the developing country since optimal level of reserves provides confidence to the markets. In this respect, they have derived specifications that include variables representing reserve depletion and opportunity cost of reserve holdings (interest rate is taken to be the representative). They have also found that during crises periods, it becomes difficult for the countries to hold optimal reserves. The results were satisfactory regarding the optimal size of reserves; but, they claim that variance in reserves can only partially be explained.

In this framework, our research focuses on the explanation of the time variation in the reserves and level of reserves during the crises periods. In the meantime, we will try

to model the volatility and forecast it. We may claim that in an emerging market, the level of international reserves is mainly determined by the volatility in the financial markets especially in the foreign currency markets. This point is important since we take into account of the motive for correcting the volatility in the foreign exchange markets.

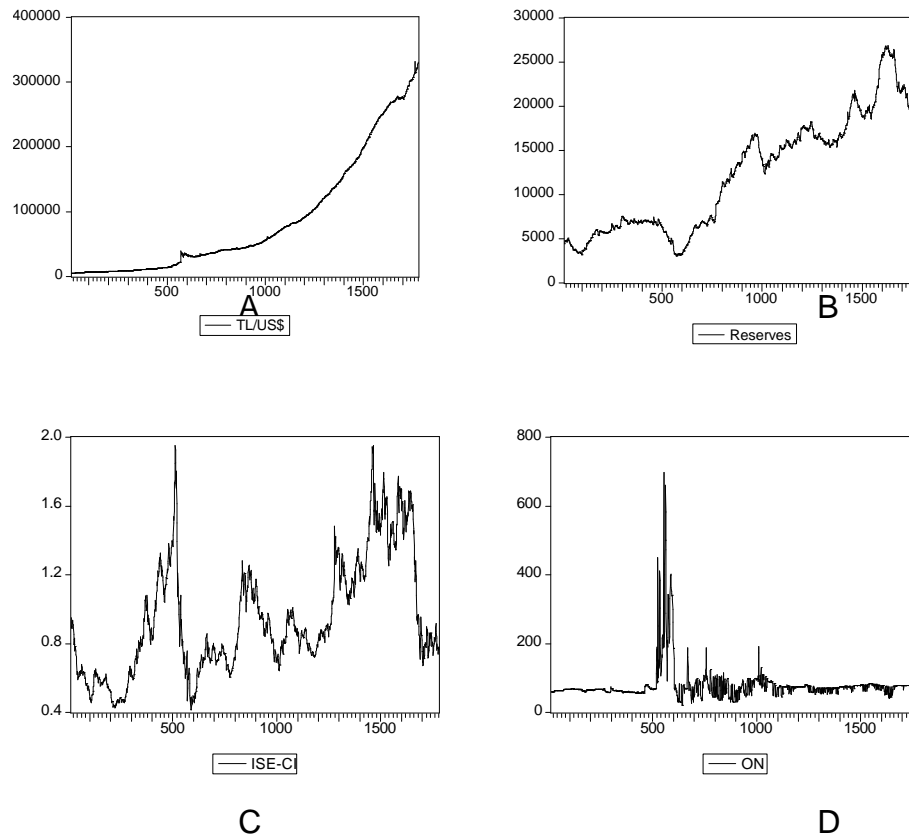
The next section will discuss the data and methodology. Section 3 presents the empirical results in the modeling stage. Section 4 is devoted to volatility forecasting which is a measure of forecast performance of the model and Section 5 summarizes the main findings and discusses the areas of further research.

## **II. DATA AND METHODOLOGY**

Daily data, excluding weekends and holidays are obtained for January 2, 1992 to January 31, 1999, including 1780 data points. The data set consists of the following items: The international reserves (reserves) of the Central Bank denominated in US\$, the TL/\$ exchange rate in nominal terms, the overnight (ON) simple interest rates obtained from the Central Bank of the Republic of Turkey database and converted into daily rates, The Istanbul Stock Exchange (ISE) composite index (CI) which is the weighted average of the closing rates and obtained from the ISE weekly bulletins and converted to US\$ units. You may find the graphs of each data below. The impact of the 1994 and 1998 crises can easily be seen in all four data sets. Moreover, increasing volatility is clearly observed in the overnight rates after the 1994 Turkish

financial market crises, whereas it is the nature of the exchange rate and stock market index data.

**Figure – The data**



This study attempts to model the time variation in the Central Bank reserves. A good candidate for such a work is the conditional volatility models of Engle (1982) and Bollerslev (1986). These models employ the volatility clustering which helps to determine the magnitude but not the sign of the shocks [large (small) percentage changes are followed by large (small) percentage changes].

It is found by many authors, that high frequency financial and economic data present, excess kurtosis as

compared to the normal distribution, time variation and positive autocorrelation (that causes problems in prediction) between the squared data that follow a decreasing pattern with increasing time lags (Heynen and Kat, 1994). These properties of the data are successfully captured by Autoregressive Conditional Heteroscedasticity (ARCH) and ARCH-type specifications since they incorporate the changing volatility in the system.

Autoregressive conditional heteroskedasticity (ARCH) was first explained in a seminal paper by (Engel, 1982) as the specification capturing the time variation in the data. The specification is as follows

$$y_t = x_t' \beta + \varepsilon_t \quad t=1, \dots, T \quad (1)$$

where  $x_t$  is a  $k \times 1$  vector of predetermined variables that includes the lag values of the dependent variable  $y_t$  and  $\beta$  is the parameter vector which is  $k \times 1$ . Let,

$$\varepsilon_t | \mathcal{F}_{t-1} \sim N(0, h_t) \quad (2)$$

conditional on the set of realized variables

$$\mathcal{F}_{t-1} = \{y_{t-i}, x_{t-i}\} \quad \text{where } i=1, \dots, n.$$

so that the variance can be modeled as

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \dots + \alpha_q \varepsilon_{t-q}^2 \quad (3)$$

$\omega > 0$  and  $\alpha_i > 0$ , which is a sufficient condition to ensure positive variances and  $\sum_{i=1}^q \alpha_i < 1$ ,  $i = 1, \dots, T$  is the necessary condition for the system to be stationary.



The model formulates the shocks to the system where a large shock is represented as a large deviation of  $y_t$  from its long-run equilibrium value (mean), which is equivalent to a large positive or negative value of  $\epsilon_{t-1}$ , so that  $q$  represents how long a shock persists. If  $q$  is large (small) then the episode of volatility tend to be long (short).

It was found by Engle (1982) that large values of  $q$  were needed to capture the episode of variances. To counteract this, Bollerslev (1986) developed a more parsimonious representation of the ARCH modeling which he called Generalized ARCH (GARCH) specification. In this representation the variance is written as

$$h_t = \omega + \alpha(L) \epsilon_t^2 + \beta(L) h_t \quad (4)$$

where  $L$  is the lag operator and  $\alpha(L)$  and  $\beta(L)$  are the lag polynomials with orders  $p$  and  $q$ , respectively; if  $1 - \alpha(L)$  and  $1 - \beta(L)$  have roots which lie outside the unit circle an ARCH representation can be written.  $\omega > 0$ ,  $\alpha_i > 0$  and  $\beta_j > 0$ ,  $i = 1, \dots, T$  are sufficient conditions to ensure positive variances and similarly  $\alpha_i + \beta_j < 1$  is necessary to ensure stationarity in the system.

For some financial data, symmetric representations as such mentioned above are inappropriate to model volatility since they fail to include the asymmetric responses to the shocks which are called leverage effects. In this case, negative and positive innovations have different impacts on volatility which is of particular importance. Thus, in modeling the exchange rate we will search for the responses of our variables

to negative and positive shocks. Nelson (1991) developed the Exponential GARCH (EGARCH) representation that takes into account the leverage effect. In this respect, he formed the following logarithmic representation which avoids non-negativity restrictions and produces dependence between the  $h_t$  and the past errors  $\varepsilon_{t-i}$ , so that if we define

$$\varepsilon_t = \eta_t \sqrt{h_t} \quad \text{where} \quad \eta_t \sim \text{i.i.d } N(0,1)$$

the variance formulation in (4) can be written as

$$\text{Log}(h_t) = \omega + \alpha(L)g(\varepsilon_t) + \beta(L) \log(h_t) \quad (5)$$

Again  $\alpha(L)$  and  $\beta(L)$  are the lag polynomials with orders  $p$  and  $q$ , respectively and

$$g(\varepsilon_t) = \gamma_1 \varepsilon_t + \gamma_2 \varepsilon_t^2 - E \varepsilon_t^2 \quad (6)$$

The asymmetry in the conditional variance is obtained from  $\gamma_1(\gamma_2 + \gamma_1)$  since (5) is linear in  $\varepsilon_t$  when  $\varepsilon_t$  is positive and  $\gamma_1(\gamma_2 - \gamma_1)$  when  $\varepsilon_t$  is negative.

The first term in (6) represents the correlation between the future covariance's and the error and second term provides the ARCH phenomenon. For stationarity the necessary condition is the  $\sum \alpha_i < 1$ .

A general approach to the estimation of the ARCH regressions is to use the Maximum Likelihood (ML) estimation as it provides consistent and more efficient estimates than OLS (Hsieh, 1989). ML can accommodate positivity restriction fairly easily (we have to secure that we have positive

variances) which is difficult in OLS. Moreover, mean and variance functions can be jointly estimated. We assume that the conditional distribution of the data has the standard normal density, though, this raises the possibility of misspecification. The general form of the log-likelihood function is as follows

$$l_t = -c - (1/2) \ln h_t - \ln f(\hat{y}_t / \hat{h}_t) \quad (7)$$

where  $\hat{y}_t$  is as given in equation (1),  $\hat{h}_t$  is as given in equations (3), (4), (5) and the parameter vector  $\theta = (\theta_0, \theta_i, \theta_j)'$  where  $i=1, \dots, p$  and  $j=1, \dots, q$  which constitutes  $(p+q+2)$  dimensions.

The identification of the lag-length in the system is done by using Likelihood-ratio (LR) test, Akaike's (1974) information criterion (AIC) and Schwarz's (1978) information criterion (SIC) as well as taking into account the tests for autocorrelation with Ljung-Box Q-statistics and Engle's ARCH-LM test in the data. The Q-statistics is a test statistic for the null hypothesis that there is no autocorrelation up to order  $k$  and is asymptotically distributed as  $\chi^2$  with degrees of freedom equal to the number of autocorrelations.

The standardised residuals are used in the diagnostics of the identification process and are represented as  $z_t = \hat{y}_t / \hat{h}_t$  which has mean zero and variance one.

**TABLE 1 – Summary statistics for the data**

	R	ER	X	ON
Mean	0.079219	0.234589	-0.005033	-6.146316
Median	0.071995	0.191711	0.024614	-6.136506
Maximum	14.73907	32.85110	16.04305	-6.135746
Minimum	-9.305999	-9.407133	-25.37675	-6.325157

Std. Dev.	1.549131	1.275979	3.241092	0.028561
Skewness	1.851548	12.37437	-0.773255	-4.435538
Kurtosis	20.32528	303.1302	9.430237	25.05773
Jarque-Bera	23266.22	6722442	3242.201	2967.508

In Table 1, some summary statistics are presented. R, ER and X are the log-first differences of reserves, exchange rate and stock market index, respectively. The ON interest rate is simply the log of the series.<sup>3</sup> These statistics exhibit highly skewed and leptokurtic behavior which are the signs of heavy-tailed distributions observed in high frequency data (Heynen and Kat, 1994)<sup>4</sup>. Exchange rates and overnight rates even present more kurtosis and skewness than the reserves and the ISE-CI. It is also obvious that the variables are not normally distributed since the Jarque-Bera normality test is rejected at 1 percent significance level in each case. The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. The 5 percent significance's critical value is 5.99 for a  $\chi^2_2$  distribution. It is seen that all the calculated values test are far greater than the critical values that rejects normality.

Tables 2 reports the Q-statistics for testing autocorrelation in levels and squares of R, ER, X and ON. The critical value of the statistics at  $\chi^2(15)$  is approximately 25 at 5 percent significance level. The Q-statistics are calculated at autocorrelation lag of 15. In the raw data, correlogram of the variables present some autocorrelation for the whole sample. However, ON rates exhibit higher autocorrelation at both level

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<sup>3</sup> We have checked for unit root in the raw data that is logarithmically scaled. We have found unit-root in reserves, stock market index and exchange rates. First differencing is applied since it is a solution for unit root.

<sup>4</sup> Normal distribution has skewness of zero and kurtosis of 3.

and squares at 1 percent level. There is evidence of autocorrelation in the squared series since Q-statistics do not reject the null hypothesis of squared autocorrelation at 1 percent significance level.

**TABLE 2 – Autocorrelation of level and squared data**

Autocorrelation coefficients Q(15)*	R	ER	X	ON
9201-9901**	69.9	359.2	65.0	11964.0
9201-9901***	33.5	310.4	710.2	11214.0
9201-9312***	4.9	39.3	15.1	4155.4
9401-9409***	37.5	30.9	74.3	1497.4
9410-9807***	3.8	60.7	134.8	1688.1
9808-9901***	31.6	32.7	13.6	94.8

\*significance at 5 percent level, \*\* levels, \*\*\* squared

We divided the sample into 4 subsamples representing the pre 1994 crises, 1994 Turkish financial crises, post – 1994 to August 1998 Russian crises and Russian crises and afterwards. It is interesting to see that autocorrelation in the squared data is present especially in the crisis periods for reserves. Exchange rates and overnight interest rates have autocorrelation in the squared data in all periods. However, the stock market index does only have autocorrelation in the squared data in the 1994 crises but not in the 1998 Russian crises. Though, we would like to take into account the autocorrelation of the squared data in the full sample period since we believe that the estimated equations should include the crises periods and the information provided by them. This will be particularly important in forecasting. For a Central Bank that has a precautionary motive it is important to monitor the increase in the volatility. In this respect, policy makers can determine the level of reserves in advance and take the

necessary measures to stabilize the volatility in the financial markets.

### **III. EMPIRICAL RESULTS**

The theories on capital flows mainly focuses on the interest rate differential between the countries. International investors invest in the countries where the returns are high. In emerging markets, returns are higher than the developed markets in order to attract foreign capital<sup>5</sup>. Eventually, increasing capital flows to the country brings an equivalent demand for the domestic currency which is usually provided by Central Banks. This obviously increases the level of Central Bank reserves.

In order to explore the dynamics of the data further and to see its characteristics, we univariately model each data. The full sample period is taken into consideration. In this respect, GARCH is used due to the fact that the procedure imposes higher weights to the last observations and lower weights to the past observations, so that information provided by earlier observations are taken into considerations with less weights than the later ones. This is due to the fact that we have to learn from the crises since these periods have weights which are small but are large in magnitude so that they affect the whole process. This accounts for two crisis, one in 1994, mainly because of domestic factors, and the other in 1998, as a result of the spillover effects of the Russian crisis.

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<sup>5</sup> We do not mention other factors, such as the risk premium rising from the risk of default, convertibility, inflation, floating exchange rates, etc. since they are beyond the scope of this research.

We have to consider some points before we move onto the univariate modeling of the reserves. Reserves do not present a heteroscedastic behavior except during currency crises (Table 2). They are relatively stable as a result of rare intervention in the foreign currency markets. Moreover, in conventional models of leading indicators for financial crises, changing expectations cannot be forecasted. These models focus on the difficulty of determining the exact starting point of the "altering expectations" that is expected to result in increasing volatility.

The framework is built on the GARCH specification since it provides a more parsimonious model. The mean equation of each variable is an autoregression of order  $m$ ,  $AR(m)$  and represented as given in equation (1) and the variance equation is represented as given in equation (4). Lag-length selection is made jointly by the LR, SIC, AIC and ARCH-LM tests. However, we are using the Bollerslev and Wooldrige (1992) robust standard errors and covariance, so that the first two moments are correctly specified, and the parameters are consistently estimated even in the case of the violation of the assumption of conditional normality. On the other hand, we believe that univariate modeling is better than multivariate modeling since problems of non-convergence and difficulties in calculating robust standard errors (Berument and Malatyali, 1998) is common in the multivariate case.

The next step in modeling univariate variances is to introduce the Central Bank's policy variable, which is the overnight rates, into equations with a one period lag. This is based on the fact that markets respond to such policies with a

one period lag. Our calculations support this phenomenon below. We believe that Central Bank policies effect the volatility in the money and equity markets. The expected sign of the ON interest rate on the stock market and the exchange rate is negative since the interest rate constitutes the opportunity cost of holding reserves. However, the interaction of the ON rates and the reserves is already a matter of dispute. During the crisis periods, the natural positive link between the interest rates and the capital flows breaks down. An increase in interest rates should invite capital inflows to the country since yields are higher. However, with decreasing confidence in the crisis periods, capital outflow is the first outcome since investors would prefer safe but low yields compared to risky and high returns. The excess demand for capital outflow brings excess demand in the foreign exchange. This creates tightness in the domestic currency markets that increases the interest rates. Nonetheless, increase in the interest rates does not promote capital inflow due to uncertainty (Binay and Salman, 1998). In this respect, Turkish data covers the two crisis periods, one depends on domestic and the other on external factors. In addition, if we believe that the effects of the crises are significantly reflected in the overall sample, then a negative relation between the reserves and the interest rates is expected<sup>6</sup>.

Since we believe that interest rates have direct impact on other markets, we have included ON in the mean equation

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<sup>6</sup> Policy makers may claim that this relationship may turn to positive above a certain threshold. We can still believe that, for a longer range, this negative relationship does hold, but for certain times the belief in the existence of a threshold may force the relationship to be positive.



and find it statistically significant. We found that GARCH(1,1) describes the univariate modeling of the reserves quite satisfactorily with two lags of the reserves in the mean equation. Results are given in column 1 of Table 3. Both lags of growth rate of reserves turn out to be significant at 1 percent level. This points that approximately a total of 0.2 percent of memory is present in the reserves where 0.10 percent coming from the first and 0.09 percent from the second lag of reserves.

**TABLE 3 - Empirical results of the modeling**

	$R_t$		$ER_t$		$X_t$
		<i>Mean equation</i>			
C	-1.893078 (0.975859)	**	-4.096058 (1.962990)	**	0.030755 (0.061364)
$R_{t-1}$	0.104666 (0.034983)	*			
$R_{t-2}$	0.093926 (0.029554)	*			
$X_{t-1}$					0.097440 (0.032936) *
$ON_{t-1}$	-0.319161 (0.154715)	**	-0.693507 (0.313664)	**	
<b>ARCH-LM</b>	1.054408		0.298561		5.688424
<b>5-lags</b>	[0.958059] <sup>c</sup>		[0.997670]		[0.337726]
		<i>Variance equation</i>			
$\omega_0$	0.126981 (0.057151)	**	-0.393804 (0.079473)	*	0.304715 (0.093584) *
$e^2_{t-1}$	0.053984 (0.020769)	*			0.124193 (0.024903) *
$h_{t-1}$	0.892720 (0.036560)	*			0.850411 (0.025875) *
$\text{Log}(h_{t-1})$			0.911143 (0.029423)	*	
$\omega_t$			0.544474 (0.117732)	*	
$\omega_t$			0.184474 (0.121108)		
$\omega_1 + \omega_1$	0.945				0.974
Log-likelihood	-3192.753		-1656.987		-4376.067
AIC	3.601298		1.870626		4.928084
SIC	3.622895		1.889130		4.943504
Q(15)	3.7848		0.9091		12.167

<sup>a</sup> The estimated coefficient <sup>b</sup> Bollerslev and Wooldrige (1992) robust standard errors <sup>c</sup> probabilities. \*, \*\* and \*\*\* indicate significance at the levels of 1 percent, 5 percent and 10 percent, respectively.

The coefficient of the  $ON_{t-1}$  turns out to be negative, this does not follow the generally accepted view of an increase in domestic interest rates are followed by capital inflows as a result of increasing margin between the foreign and domestic rates. However, the Central Bank is using the interest rates to overcome the losses in the foreign reserves during capital flows and the expected sign of the interest rate should be

positive, so that, the Central Bank policy aiming at halting the fall in the reserves by increasing interest rates does not work. Rather the interest rate may play the role of opportunity cost of holding reserves or a certain threshold is present but is not represented in our specification. It is also evident that countries that were in financial crises tried to use the interest rates to stop the loss in reserves. Still, none of them succeeded and eventually devalued their currencies. In our findings, we see that an increase in the interest rates by one percent decreases the reserves by almost 0.3 percent. The Q-statistics does not present autocorrelation in the squared residuals with a calculated value of 3.78 and ARCH-LM statistics does not present misspecification with a p-value of 0.96.

When we examine the variance equation, we find out that GARCH effects are present and significant. The necessary and sufficient conditions are satisfied where both  $\alpha_1$  and  $\alpha_2$  are between zero and one and sum-up to 0.945 which ensure positive variance and stationarity, respectively. The Q-statistics on standardized residuals calculated at order 15 presents that no autocorrelation is present at the squared residuals.

The second column of Table 3 presents the results of the modeling of exchange-rate volatility. Since ARCH has been used as an effective specification for estimation, various formulations were observed. Pioneering studies began with ARCH (Domowitz and Hakkio, 1985) but further studies employed GARCH such as Bollerslev (1987), Baillie and Bollerslev (1989), Xu and Taylor (1995) and McKenzie (1997) because of the efficiency gained by using smaller number of variables. With the introduction of EGARCH specification, it is

observed that asymmetry in the exchange rates is easily captured (Hsieh, 1989), (Andersen and Bollerslev, 1998) and (Sengupta and Sfeir, 1996).

The EGARCH specification that we use in this study differs in some points from the original approach that Nelson (1991) suggested. First we assume that  $\epsilon_t$  is distributed normally and the estimated constant at the variance function differs from the real constant by  $E\epsilon_t^2$ . There is the evidence of the negative “leverage effect”, since the coefficient  $\beta$  is positive and statistically different from zero. Also, the EGARCH term is statistically significant at 1 percent level. This finding shows that negative and positive shocks are treated differently. This means that an increase in the speed of depreciation of the TL against the US dollar quickens the depreciation further as compared to the appreciation, so that, a shock of appreciation creates less volatility than a shock of a depreciation that creates an upward bias in the depreciation of the currency. This reflects that a shock towards depreciation needs higher intervention in the foreign currency markets and acceleration in the loss of reserves. Consequently, reserve volatility increases.

It is interesting to see that in the exchange rate equation, overnight interest rate turns out to be significant, however, the coefficient of the lag of the exchange rate is found to be insignificant. ARCH-LM test does not show the presence of misspecification with a p-value of 0.99. The coefficient of the  $ON_{t-1}$  is negative and significant where one percent change in the overnight rates today decreases the change in the devaluation (revaluation) by 0.7 percent

tomorrow. This reflects the fact that players in the market make corrections in every period since they believe that the Central Bank would eventually intervene in the markets to make the necessary correction. This is a sign of dominance of the Central Bank in the foreign exchange markets.

The coefficient in front of EGARCH is less than one which brings finite conditional variance. The existence of positive "leverage effect" is again supported. The Q-statistic calculated by using the standardized residuals at 15<sup>th</sup> order is 0.9 which is significantly lower than the critical value of 25.

The next step in our research is to model the volatility of ISE-CI. Earlier studies find that GARCH in means that is GARCH-M(1,1) presents satisfactory solutions for the modeling of return in the stock markets (Chou, 1988), (Susmel and Engle, 1994) and (Choudry, 1996) and evidence for the Turkish case was examined and found in Salman (1999). Empirical evidence also indicates that there is a significant day-of-the-week effect in the ISE, as found by Balaban (1995) who included the day-of-the-week effect dummies in both the return (mean) and risk (variance) equations of the model. However, inclusion of such variables did not contribute to our findings in the full sample period, so that we do not include them in our calculations. In modeling volatility, we found that the model that best describes the return for the full sample period is GARCH(1,1). The results are given in column 3 of Table 3. It is clearly seen that GARCH(1,1) fits the data very well with one-lag of the dependent variable. Since ON interest rates are not significant we did not include them in our specification. It is found that 1 percent increase (decrease) in

return at this period increases (decreases) return in the following period by 0.1 percent. The system is stationary and has a positive variance since the sum of the ARCH and GARCH effects is 0.974 which is less than one and both of the coefficients are positive.

One of the important findings of our specification is the significantly low value in the correlogram of squared standardized residuals. This also supports the use of ARCH-type specification in estimation. The Q-statistics at 15<sup>th</sup> order autocorrelation is 3.8 for the full sample. Similarly, in exchange rates and stock market index, the Q-statistics are 2.3 and 12.2, respectively. Besides, the ARCH-LM test calculated at 5 lags does not present misspecification at p-values of 0.95, 0.99 and 0.33 for the reserves, exchange rate and stock market index, respectively.

The next step is to search for the volatility in the international reserves taking into account the volatility and percentage changes in the other markets. In this respect, we will introduce the conditional variance, return (percentage change in the stock market index) and exchange rate in the mean and variance equations and see the effects of it on reserves. We are aware that inclusion of variances in the reserve equation creates problems in calculating robust standard errors, on the other hand, we are not taking account the possible feedback's. Therefore, we can still suspect the inefficiency in calculating the t-statistics. However, we still believe that this model works better than the multivariate model due to the belief that tackling with that problem creates further problems, as in the case of Berument and Malatyali (1999). On

the other hand, the ARCH-LM test still supports the absence of misspecification. The results are presented in Table 4.

**TABLE 4 - Empirical results of the volatility interaction in modeling**

	$R_t$		$R_t$		$R_t$		$R_t$
	<i>Mean equation</i>						
$h_t$							-0.023229 (0.030909)
C	-2.187841 <sup>a</sup> *		-1.769631 *		-2.246423 *		-1.693887 *** (1.008057)
$R_{t-1}$	0.100738 *		0.099806 *		0.099941 *		0.096928 * (0.038294)
$R_{t-2}$	0.096339 *		0.095405 *		0.095348 *		0.095538 * (0.029970)
$ON_{t-1}$	-0.358645 *		-0.299303 **		-0.366155 *		-0.294424 *** (0.158953)
$X_t$	0.039757 *		0.038878 *		0.038725 *		0.041042 * (0.011758)
VolX	0.004845 (0.007214)				0.007659 (0.008554)		
VolER			-0.0000487 (0.0000165)	*	-0.0000695 (0.0000324)	**	
<b>ARCH-LM</b>	1.006894		1.011208		1.005704		1.104116
<b>5-lags</b>	[0.962008] <sup>c</sup>		[0.961657]		[0.962104]		[0.953739]
	<i>Variance equation</i>						
$\gamma_0$	0.131793 **		0.132919 **		0.132623 **		0.123389 ** (0.065434)
$\gamma_{t-1}^2$	0.062934 *		0.059898 *		0.062887 *		0.067554 * (0.026575)
$h_{t-1}$	0.883423 *		0.885217 *		0.883001 *		0.885682 * (0.044404)
$\gamma_1 + \gamma_{t-1}$	0.945		0.941		0.945		0.952
Log-likelihood	-3186.216		-3186.611		-3185.362		-3196.902
AIC	3.596191		3.596636		3.596356		3.608219
SIC	3.623960		3.624404		3.627209		3.635987
Q(15)	3.5206		3.5285		3.5222		4.0462

<sup>a</sup> The estimated coefficient <sup>b</sup> Bollerslev and Wooldrige (1992) robust standard errors <sup>c</sup> probabilities. \*, \*\* and \*\*\* indicate significance at the levels of 1 percent, 5 percent and 10 percent, respectively.

The first column of Table 4 summarizes the relationship between the volatility in the stock market, return and the reserves through mean equation<sup>7</sup>. Our variables are VolX and X; we may expect a negative sign of the coefficient of the

<sup>7</sup> We have also tried volatility in the variance function however, coefficients did not come out to be significant and coefficients in front of the variance and the error altered, which is not expected.

volatility of stock market and positive sign for the index. The reasoning is as follows; a volatility increase should attack the confidence in the markets, and spillover effects are observed in other markets. The stock market index is the reflection of the general confidence of the investors to the markets conditions.

We observe that volatility in the stock market is positive but also insignificant. This implies that sharp changes in the stock market index which do not have significant impact on the growth rate of reserves. We may claim that most of the sharp changes in the stock market are not as a result of the behavior of foreign investors. So that every capital outflow from the stock market would not result in an increasing demand for foreign exchange.

In the mean time, return is positively and significantly (at one percent level) related to the growth rate of reserves. This implies that when confidence in the financial markets is strong then demand for foreign exchange declines and vice versa. In this respect, a one percent increase (decrease) in return result in a 0.04 percent increase (decrease) in the growth rate of reserves. It is observed that the overnight interest rate is still negatively related to the growth rate of reserves, so that one percent increase in the overnight interest rate decreases the growth rate of reserves by 0.35 percent. The stationarity holds since the sum of the coefficients in the conditional variance is 0.947 which is less than one and all coefficients are positive. ARCH-LM test does not present the presence misspecification at p-value of 0.95. This is also supported by the Q-statistic since 3.7297 is far less than the critical value of 25. The coefficients in the mean and variance



equations do not alter significantly. Within this framework, we are still on the safe side and can conclude that the volatility in the stock market does not affect the reserves.

The second column represents the interaction of the volatility in the exchange rate (VolER) and return to reserves. As indicated above an increase in the exchange rate volatility brings frequent intervention in the foreign exchange markets by the Central Bank. The negative sign also supports this hypothesis; an increase in the volatility of the exchange rates result in the decline in the level of reserves. Although the magnitude is small (-0.0000487), significance is achieved at 1 percent level. It is also seen that some part of the information provided by the overnight interest rates are captured by the exchange rates but no significant change is seen in the coefficient of return. We may conclude that a one percent increase in the overnight rates decreases the growth rate of reserves by 0.3 percent. Moreover, the sign and significance of the coefficient of the return do not change. Misspecification is rejected at a p-value of 0.96 by ARCH-LM statistics.

In order to see whether both the volatility in the stock market and the exchange rates affect reserves jointly or not, we ran the specification that is presented on Column 3, however, no significant improvement is observed. The last point to note is that we also suspected the interaction between the volatility of the reserves which is the GARCH-M specification and itself. In this respect, we include the conditional variance of reserves included in the mean equation. We see that reserve volatility is negatively related to the growth rate of reserves; however, we could not find any

evidence of such a relationship as in the case of the stock market volatility since the coefficient is not significant even at 10 percent significance level. And finally, in four of the equations, we see that positive variance and stationarity conditions are satisfied; all the coefficients are greater than zero and the sum of the coefficients are 0.945, 0.941, 0.945 and 0.952, which are evidently less than one<sup>8</sup>.

#### **IV. VOLATILITY FORECASTING**

A substantial literature is devoted to forecasting since one way of measuring the performance of a model is estimating its forecasting power. In this framework, volatility forecasting is used to assess the volatility prediction model's ability to predict future volatility's. As found by Noh, Engle and Kane (1994) GARCH volatility estimates and forecasts perform better than other volatility models. They are sufficient in capturing the structural changes in the time series.

The GARCH model forecasts future volatility by using the return series of the variables. We use the specification as shown in (1), (4) and (5) and estimate and forecast volatility in each day with the available data. As stated earlier maximum likelihood estimation is used. The previous day values are used to be the initial values in forecasting, so that the updated information set is used to forecast the volatility for an arbitrary length of time. We believe that one week forecasting horizon would be satisfactory for a Central Bank to forecast the growth

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<sup>8</sup> One may claim that the sum of the coefficients in the variance equation is very close to one, and nonstationarity may arise in the specification. We may say that it is correct, however, empirical work beginning with Engel (1982) frequently finds values very close to one.

rate and the volatility of reserves in order to get the necessary measures to continue with the monetary policy goals in advance.

The results that are calculated in the previous section led us to choose the specification that is represented in column 2 of Table 4. For out-of-sample forecasts there are various statistics to measure the performance of the specification. In this respect, we use the difference between the realized returns and estimated parameters to calculate the forecast errors.

The forecast variability is measured by the forecast standard errors. Suppose the forecast sample is "t = s, s+1, ..., s+h" and denote the actual and forecasted value in period t as  $y_t$  and  $\hat{y}_t$ , respectively. The statistics "Mean Squared Error (MSE)" and "Root Mean Squared Error (RMSE)" that will be presented in Table 5 are represented by equations (8) and (9)

$$MSE = \frac{1}{h} \sum_{t=s}^{s+h} (y_t - \hat{y}_t)^2 \quad t = s, \dots, s+h \quad (8)$$

$$RMSE = \sqrt{\frac{1}{h} \sum_{t=s}^{s+h} (y_t - \hat{y}_t)^2} \quad t = s, \dots, s+h \quad (9)$$

The MSE forecast errors depend on the scale of the dependent variable so that forecast performance of different models can be compared. As the error gets smaller, the forecast performance of the model increases.

The period of April 1<sup>st</sup>, 1998 ? January 31<sup>st</sup>, 1999 is chosen to assess the forecast performance. The rationale behind choosing this period is its inclusion of the Russian crises in August 1998. In this respect, by using the information

at the 1994 crisis, we will be able to see the performance of our volatility forecasts in capturing the Russian crisis.<sup>9</sup>

For realized volatility we calculate the average of the five day ahead errors variances, so that the MSE will be calculated by taking the difference between the forecasted variance and the realized variance. We considered four alternative volatility forecasts for forecasting volatility of the Central Bank reserves. These are, constant volatility forecast including the crises periods, constant volatility forecasts excluding the crises periods, weekly historical forecast and GARCH volatility forecast. For the first case, we simply calculate the variance of the full sample and use it as a constant parameter. For the second case, we calculate the variance in the full sample period excluding the periods of crises. In this case, we expect a lower variance since shocks are not included in the calculations. For the third case, we calculate variances beginning from the last week of the sample period and introduce the calculated variances as the forecast observations recursively. For the last case, we use the conditional variance as stated in equation (4) and for the forecasts of the conditional variance we use the model as stated in the equation (12).

Initially, we begin with the specification represented on column 2 of Table 4. We have to determine and forecast the explanatory variables first. We have to forecast the exchange rate volatility that was specified by EGARCH (1,1). To forecast we will use the specification in equation (13). The next step is

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<sup>9</sup> The specification in column 2 of Table 4 is re-estimated and the results are

determining the policy variable which is the overnight interest rates. We keep the overnight rates constant from the last point of the sample. On the other hand, the forecasts of the return are calculated by using the specification as in column 3 of Table 5. In this respect, we can calculate the out-of-sample growth rate of reserves by using the estimated parameters and the forecasted exchange rate volatility and return with constant interest rates. The difference between the calculated reserves and the realized values is the forecast error. The variance of the weekly forecast error is treated as realized variance. The results are presented on Table 8.

**TABLE 5 - Out-of-sample forecast performance (weekly)**

	<b>MSE</b>	<b>RMSE</b>
Constant volatility with crises periods	3.33	1.82
Constant volatility without crises periods	1.90	1.38
Historical	1.92	1.39
GARCH	1.83	1.35

When we examine the MSE, we see that GARCH MSE is the lowest of all with 1.83. Constant volatility including the crises periods has the highest MSE with 3.33. The second and third bests, according to MSE, are the constant volatility excluding the periods of crises and weekly historical with 1.90 and 1.92, respectively.

The difference between the GARCH and constant volatility estimates rises because of the weighting of the two different statistics. Constant volatility gives every observation the same weight, while, GARCH gives the last observations higher weights than the previous observations. Although, this is the case, the two crises (1994 and 1998) have significant

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presented in the Appendix.

impact on the overall estimates and we believe that GARCH models them successfully. On the other hand, the information set of weekly historical and GARCH are similar. The distinction is; weekly historical assumes that the values before the last five variances are all zero, so that, GARCH uses longer information that helps the specification to learn from the past. In this respect, shocks that have large impact on the system are already included in GARCH.

The results provide an answer to the explanation of variance in reserves which was partly explained in Ben ? Bassat and Gottlieb (1992) and prevent the sudden reserve depletion since the level of reserves are adjusted to the crisis periods, so that, a high level of reserves will decrease the variability as well as build the investors confidence in the financial markets.

## **V. CONCLUSION AND FURTHER RESEARCH**

The main idea of the research is econometric modeling of the dynamics of the Central Bank reserves in an emerging market setting in an attempt to determine the required level of reserves. In this respect, we have followed the usual procedures beginning from the data analysis and choosing the ARCH specification. In the modeling stage, effects of the volatility in the stock market and foreign currency markets and stock market return on reserves are examined. Overnight interest rates are introduced in the model as exogenous policy variable of the monetary policy. It is found that overnight rates and the volatility in the exchange rate have a negative and stock market return has a positive effect on reserves. It is

interesting to see that volatility in the Istanbul Stock Exchange (ISE) does not affect reserves at all, but the stock market return has a significant effect. In this respect, we may say that it is the general confidence in the markets which is reflected by the stock market return that plays an important role on the level of the reserves.

One other way of measuring the performance of the model is to assess the predictive power of the volatility forecasts. Also, by employing this, we will be able to have foresight in forecasting the volatility. Several forecast statistics are calculated in the relevant sections. We observe that out-of-sample forecasts present satisfactory results that indicates that the model chosen has good forecasting performance. Moreover, this indicates that the policy maker has to hold more reserves in the stable periods to signal the strength and maintain stability in every period.

One important caveat that we need to mention is the use of overnight interest rates as the only policy tool of the Central Bank representing market rates. However, we know that, after 1994, especially the secondary market volume of trade increased and the interest rates in these markets diverged from the overnight rates. In this respect, a weighted average of the interest rate has to be used in the model. However, at this point the data needed for the analysis was not available.

Optimal reserve targeting literature, as we discussed above, focused on the difference between the behavior of developed and developing markets, countries inviting high and

low rates of short-term capital and countries with high and low rate of imports to reserve ratios. We believe that emerging markets need more reserves than the developed markets since hot money is more of a concern for an emerging market than a developed market, so that emerging markets may provide the investors with more confidence and lead them to change their expectations less frequently. In this respect, further research should concentrate on taking the short-term capital inflows as the benchmark for the level of reserves in addition to the volatility and import measures. On the other hand, it would be easier to determine the behavior of the capital flows and its relation to the level of volatility in reserves. In the mean time, the relationship between the reserves and macroeconomic variables should be examined, since the cost and benefits of reserve holdings can be established and interactive variables determined.



## Appendix

**TABLE A - Empirical results for growth in reserves equation  
(January 1<sup>st</sup>, 1992 ? March 31<sup>st</sup>, 1998)**

		<b>R<sub>t</sub></b>			
	<i>Mean Equation</i>			<i>Variance Equation</i>	
C	-1.854369 <sup>a</sup> (0.997567) <sup>b</sup>	*	?	0.138082 (0.066055)	**
R <sub>t-1</sub>	0.107700 (0.036616)	*	?	0.054500 (0.021585)	*
R <sub>t-2</sub>	0.088802 (0.030765)	*	h <sub>t-1</sub>	0.889011 (0.039050)	*
ON <sub>t-1</sub>	-0.314177 (0.158217)	**	?	0.943	
X <sub>t</sub>	0.038900 (0.013296)	*	1		
VolER	-0.0000510 (0.0000172)	*			
<b>ARCH-LM</b>	0.655505				
<b>5-lags</b>	[0.985321] <sup>c</sup>				
<b>Q(15)</b>	2.3713		<b>AIC</b>	3.637698	
<b>Log-likelihood</b>	-2844.774		<b>SIC</b>	3.668433	

<sup>a</sup> The estimated coefficient <sup>b</sup> Bollerslev and Wooldrige (1992) robust standard errors <sup>c</sup> probabilities. \*, \*\* and \*\*\* indicate significance at the levels of 1 percent, 5 percent and 10 percent, respectively.

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