Optimal Monetary Policy, Commitment, and Imperfect Credibility*

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Abstract

In the conventional optimal monetary policy framework, two key assumptions underline the full commitment solution: Monetary authority is perfectly credible, and can commit for an infinite number of periods. Using a baseline forward looking model, this study explores the implications of relaxing these assumptions in turn. First, finite lasting commitments are introduced using a stochastic exogenous process that generates policy reoptimizations. As a consequence, monetary policy is characterized with a continuum from pure discretion to full commitment. Second, we solve the optimal and robust targeting rules when the central bank confronts imperfect and/or uncertain credibility. Imperfect credibility is defined as a situation in which the private sector expects the commitment regime to end sooner than that is intended by the policy maker. The results indicate that, under imperfect credibility, optimal policy becomes observationally closer to the discretionary solution, the more being so as the degree of uncertainty rises. These findings may be insightful for explaining the observed near-discretionary behavior of the central banks, which indeed operate under imperfect credibility.

Keywords: Optimal monetary policy; stabilization bias; imperfect credibility; discretion, commitment.

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1 Introduction

After the introduction of New Keynesian Phillips Curve, an enormous number of academic papers have inquired into the nature of optimal monetary policy under forward looking models. The analysis often centered on a binary mode, that takes the form of discretion (period by period optimization) or commitment (rule like policy under perfect credibility), in which relative advantages of one mode on the other is contrasted.[1] Similar ideas have played a pivotal role in the practical policy discussions as well.

In this literature, optimal monetary policy under full commitment is derived under the assumptions that the central bank is perfectly credible and policy maker optimizes once and for all to commit to a state contingent rule, which is meant to last forever. On the other hand, optimal policy under discretion is defined as optimizing every period where the private agents do not expect the policy to last more than one period.

Ironically, neither of the two modes of central bank behavior has been firmly supported by theoretical or empirical studies.[2] This observation suggests that, from a positive standpoint, it may be insightful to deviate from the binary treatment. In this paper, we present an analytically tractable modification of the conventional baseline optimal monetary policy formulation to allow for such a deviation. Accordingly, this study provides a general framework in which the policy maker can pursue discretion, commitment or somewhere in between, which we call "imperfect commitment". Therefore, unlike the methods in the previous literature, our technique does not draw a thick line between discretionary and commitment solutions.[3]

What do we mean by imperfect commitment? In the conventional optimal monetary policy framework, there are two key assumptions underlining the full commitment solution. First, central bank optimizes once and for all; that is, the commitment lasts forever. Second, policy maker is perfectly credible, i.e., private agents' expectation of future policy is consistent with true central bank behavior. By imperfect commitment we mean relaxing either of these assumptions in turn, first, by assuming that there is an exogenous stochastic process

[1] See e.g. Clarida Gali and Gertler (1999), Galí (2001) and Walsh (1998).

[2] See McCallum (1999) for a thorough discussion.

[3] One exception is "optimality from a "timeless perspective" that is proposed by Woodford (1999). That is, each period's is conducted as if current macroeconomic conditions are not known; as if the optimization had been made in the distant part. In this case the discretion and commitment delivers the same solution.

generating the timing of re-optimizations (finite lasting commitment), and second, by allowing the beliefs of the private agents to differ from that of the true central bank intentions (imperfect credibility).

Accordingly, we present two (not necessarily independent) consecutive setups for the analysis of imperfect commitment. The first one draws upon the so called quasi-commitment concept and the related solution method developed by Schaumburg and Tambalotti (2001) based on the work by Roberds (1987), in which both the private agents and the central bank expect, each period, a re-optimization—namely, a regime change—to occur with a fixed probability known to both parties.[4] Therefore, in general commitment is expected to last for a finite number of periods. Consequently, degree of commitment is represented with a continuum from 0 to 1, which nests discretion and full commitment as two ends of the spectrum. This case corresponds to a situation in which policy reformulations are generated by institutional or "natural" factors limiting the ability of the policy maker, independently from the intentions of the policy maker. Therefore, average regime duration that the monetary authority has on mind is exactly the same as private sector's.

The second set up completes the imperfect commitment analysis by relaxing imperfect credibility assumption—modeled as a situation in which the private sector expects the commitment to last shorter than actually intended by the policy maker. The difference between the private sector's expectations and the true policy ability is then used to construct a measure of credibility. This case can arise when e.g., average duration of commitment regime increases due to a stabilization program, institutional and/or governor-specific adjustments, creating a wedge between the private agents' and central bank's subjective beliefs. It is shown that, in such a situation, an optimizing monetary authority will choose a less history dependent rule than the case of perfect credibility.

Moreover, we argue that imperfect policy credibility is incomplete without an analysis of uncertainty. Accordingly, we perform a joint analysis of imperfect and uncertain policy credibility where the central bank confronts multiple priors over private sector's beliefs. While formulating the policy, the central bank is assumed to choose a targeting rule that performs

[4] The analysis in this study differs from Schaumburg and Tambalotti's work in at least two important ways. First, unlike these authors, we focus on the behavior of the central bank and characterize the optimal policy rule in analytically. Second, we do not agree with their interpretation of the average frequency of re-optimizations as credibility, and develop an alternative definition of credibility. These authors, independently incorporated a similar analytical solution into their paper after the first version of this study was circulated.

reasonably well over a variety of private sector beliefs. As an alternative definition, we argue that, the size of the uncertainty set can be used to derive a measure of credibility. In this setup, larger uncertainty set about beliefs translates into less credible policy regime. It is shown that, under such a representation of imperfect credibility, robustly optimal behavior of the individual policy makers will be observationally closer to discretionary behavior, in the sense that higher uncertainty is associated with less history dependence.

It is worth to emphasize at this point that this paper is not about how to build a commitment mechanism, or why central banks may have imperfect credibility. The degrees of policy credibility and commitment are exogenously determined, and thus, unlike reputational models, cannot be altered by the central bank.[5] To reiterate, re-optimizations and beliefs are generated by exogenous stochastic processes. Moreover, assuming a simple process that generates the re-optimizations, and using a purely forward looking model, we are able to obtain, an analytical characterization of the optimal monetary policy rule and the impulse responses of the model to a supply shock.

Rest of the paper is organized as follows. The next section outlines the baseline structural model and objectives of the central bank. Section three characterizes the behavior of a central bank committing for finite periods. The fourth section takes up the same problem under imperfect and uncertain credibility and derives the robust optimal monetary policy. Finally, the last section concludes.

2 The Structural Model

The New-Keynesian aggregate-supply equation (AS) takes the form

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \tag{1}$$

where π_t is the period t inflation rate defined as the percent change in the price level from $t-1$ to t , x_t is the output gap which is defined as the percentage by which output exceeds its potential, β is a discount factor, κ is a positive coefficient and u_t is an exogenous disturbance term. We use the notation $E_t \pi_{t+1}$ to denote private sector expectations regarding of π_{t+1}

[5] We can justify this idea as follows. In his survey paper on credibility, Blinder (1999) argues that the practitioners of the monetary policy and the academics have agreed on the idea that credibility can be built slowly through time by a consistent track record. Motivated by these findings, we assume that the incumbent central bankers do not stay long enough to build credibility and there is no coordination between the central bankers. Therefore they take the credibility as given.

conditional on information available in period t. Equation (1) relates inflation to output gap in the spirit of a traditional Phillips curve. In contrast to traditional Phillips curve, current inflation depends on the expected future course of the economy, and thus on the expectations of future monetary policy. Within the framework, monetary policy affects real economy, because sellers cannot change their price every period as in Calvo (1983), and Yun (1996). The parameter κ can be interpreted as a measure of the speed of the price adjustment. Firms set prices based on expected marginal costs. Output gap (x_t) captures the marginal costs associated with excess demand. This specification allows for a shock u_t, which shifts the distance between the potential output and the level of output that would be consistent with zero inflation.[6] These shifts are not considered to represent variation in potential output, and thus appear as a residual in (1). In what follows, we will name u_t simply as the "supply shock".[7]

Aggregate demand (IS) equation takes the form

$$x_t = -\varphi \left[i_t - E_t \pi_{t+1} \right] + E_t x_{t+1} + g_t, \tag{2}$$

where i_t is the central bank's instrument and short term nominal interest rate, φ is a positive coefficient representing the intertemporal elasticity of substitution, and g_t is an exogenous disturbance. Deviations of output from the potential depends upon real interest rate, expected future output gap and a demand shock. Thus, output gap also depends upon expected paths of real rate and the demand shock. The shock g_t can be interpreted as an exogenous variation in autonomous expenditure.

These structural equations can be derived as log-linear approximations to equilibrium conditions of a simple dynamic general equilibrium model in which the infinitely lived representative household maximizes its lifetime utility.[8] Disturbance terms g_t and u_t follow zero mean i.i.d. processes with standard deviations of σ_g and σ_u. The two structural equations (1) and (2) together with a policy rule determine the equilibrium evolution of endogenous variables π_t, x_t and i_t.

[6] An example would be a variation in the markup over the wholesale prices.

[7] In the literature u_t is generally named as "cost push shock" (see Clarida, Gali and Gertler (1999). Giannoni (2000), in a similar framework, justifies the presence of a shock term in the AS equation, using the microfoundations and by assuming that policymaker aims at stabilizing output around some efficient level – the level of output that would prevail under flexible prices and no market power. He calls this shock "inefficient supply shock".

[8] See Woodford (2002) Chapter 4 for the foundation and a derivation of a similar model from first principles.

2.1 Objective of the Central Bank

Traditionally, researchers have assumed that objective of monetary policy is to minimize a quadratic loss that contains a weighted average of variability of output gap and inflation around corresponding targets.[9] Adhering to this tradition, we assume the following loss criterion:

$$L_t = E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 \right) \right] \tag{3}$$

where λ is the relative weight assigned to output stabilization and t is the initial date the policy is adopted.[10] Woodford (2001) shows that a similar loss function can be obtained by performing a second order Taylor approximation to the expected utility of the representative household in the model that has been used to derive the structural equations.[11] Note that the steady state of the model for both inflation and output gap is zero which coincides exactly with the bank's targets for these variables. Therefore, there is no conflict between the policymaker's objectives and the steady state. Note that this case does not have the classic time inconsistency (inflationary bias) problem that is emphasized by Barro and Gordon (1983) or Kydland and Prescott (1977), because there is no futile attempt to stimulate output above its potential level. However, as argued by Clarida, Galí and Gertler (1999) and Woodford (1999), there is still an important source of time inconsistency — namely, stabilization bias — arising from forward looking behavior. Considering only this form of time inconsistency allows us to assess gains accrued from commitment due to forward looking behavior of the model, without the need to deal issues such as differing expectations of the steady states across different regimes.

2.2 Policy Rule

The monetary authority chooses a policy rule so as to implement the equilibrium processes x_t, π_t that minimizes L_t subject to (1) and (2). In general any prescribed guide for monetary policy conduct will be regarded as a policy rule. Throughout the study, we focus on a special

[9] See Walsh (1998), chap. 8; Woodford, (2002), Clarida Gali and Gertler (1999), and Svensson 2001(b) for a recent discussion on this kind of objective function.

[10] As is shown in Woodford (2001), the welfare theoretic λ is a function of the parameters of the structural model. For our purposes, it will be sufficient to assume λ as a composite parameter that reflects simply the relative weight of the output gap in loss function.

[11] See e.g., Beningo (2002) for an open economy version of the derivation of quadratic welfare function.

type of policy rule which Svensson and Woodford (2003) call "specific targeting rules"– rules that are expressed as a direct condition for target variables, i.e., endogenous variables that enter the loss function. Later, these type of rules were called as direct targeting rules by Giannoni and Woodford (2003a).

Direct targeting rules are argued to have advantages over the general targeting rule (a high level specification of monetary policy rule that specifies the target variables, target levels and the loss function) on the grounds of higher efficiency in communicating with the public. Since they can be expressed as a first order condition, direct targeting rules will also be convenient for analytical representation of the optimal monetary policy. On the other hand, as argued by Svensson (2002, 2003), these rules are more robust to changing structure of the economy than the instrument rules expressed as an interest rate reaction function to certain variables.

Moreover, the type of rules we consider do not depend on the characteristics of the aggregate disturbances. In that sense, they are robustly optimal as defined by Giannoni and Woodford (2003a).[12] As these authors argue, restricting attention to robustly optimal rules will allow one to reach more definite conclusions about the nature of an optimal rule, since they do not depend on a specific description of the shock process.

Accordingly we will postulate that the optimal policy can be formulated as a linear relation between current, future and past target variables, i.e.,

$$P_t(..., x_{t+1}, x_t, x_{t-1,...,}\pi_{t+1}, \pi_t, \pi_{t-1}, ...) = 0.^{13} \tag{4}$$

Formally, we will denote the vector of policy rule coefficients as ψ. There is no limit on the dimension of ψ. However, in practice, we will assume that ψ is drawn from some finite dimensional space $\Psi \in \mathsf{R}^n$. Whenever the central banker re-optimizes, she chooses a policy rule $\psi \in \Psi$ to implement the equilibrium processes that minimize (3) subject to (1), (2), (4) or equivalently she chooses the processes x_t and π_t to minimize (3) subject to (1) and (2).

[12] It is important to remind here that robustness against shocks is not the only type of robustness one can consider. For example, in this study we propose a rule that performs reasonably well across a range of credibility. See e.g., Sargent and Hansen (2001) and Giannoni (2002) for other types of robust rules.

[13] As will be seen later, fully optimal rule falls within this characterization, in the model used here; thus restricting ourselves to linear targeting rules is not binding.

3 Optimal Monetary Policy under Finite Lasting Commitment

In the optimal monetary policy literature, researchers have been analyzing two ends of the spectrum : on the one end, discretionary policy where the monetary authority does not give any promises for the future and re-optimizes every period; on the other end, commitment with perfect credibility in which the policy maker optimizes once and for all and sticks to a state contingent rule forever. In reality, no central bank operates under pure commitment or pure discretion. Although it appears to be a natural step to explore, there has been surprisingly little effort on modeling the optimal policy as a continuum between discretion and commitment.

One way to deviate from the perfect commitment case is to introduce the possibility of finite durations of a commitment regime. To achieve this goal, we add a minor complication to the baseline model. As in Roberds (1987), and Schaumburg and Tambalotti (2001), it will be assumed there is an exogenous probability that the incumbent central banker may re-optimize. This may result either because of a reappointment of the central banker or because of a realization of a "large" shock. In either case the monetary authority disregards the previous commitments and sets up a new policy rule that is optimal as of the most recent period. Upon re-optimization, central bank does not switch to discretion, she commits to a new rule which is also expected to last for a finite number of periods. By introducing an exogenous stochastic process that generates the re-optimizations we are able to relax one of the key assumptions of policy making under full commitment: commitments last forever. However we maintain the perfect credibility assumption for now.

3.1 Formulation of the Central Bank's Problem Under a Continuum from Discretion to Commitment

To incorporate imperfection into the commitment process we will assume that there is an exogenous fixed probability that current commitment will not be sustained. This can be motivated by exogenous factors that forces the policy to be reformulated or central banker to be replaced. Let α denote such a probability. Each central banker can credibly commit to a targeting rule for a random number of periods ($\frac{1}{\alpha}$ on average).[14] In other words,

[14] Since α is exogenous, the policy maker cannot control the exact timing of next reoptimization.

central bankers cannot guarantee that their promises will last forever, but they make sure they will keep the promises as long as their term last. In this set up, $1-\alpha$ represents the policy maker's ability to commit.[15] The higher is $1-\alpha$, the higher is the likelihood that the central bank will stick to the commitment.

Private agents know α and form rational expectations accordingly. Therefore, although the monetary authority lacks some degree of ability to commit (with a nonzero α), she has perfect credibility of intention since the central bank's intention to commit matches exactly with her actions on average, and it is perfectly observable by the private agents.[16] Moreover, we assume that both the central bank and the private sector know the structural parameters of the model, current realization, and persistence of shocks and have the same information about the future evolution of the exogenous disturbances.

The solution can be obtained in two steps as in the pure commitment case. The first step is to compute the inflation and the output gap process that minimizes the loss function subject to (1). The second step is to implement the equilibrium processes by choosing the appropriate nominal short rate i_t. Therefore, equation (2) remains irrelevant for the determination of target variables.[17] Accordingly, from now on we will treat the price setting equation (AS) as the only constraint along with the exogenous probability of a re-optimization.

In order to gain some insight into the policymaker's problem, one can observe that it has a recursive structure and thus, each newly appointed central banker faces exactly the same type of problem. At any re-optimization period t, the policy maker will solve

$$\min_{\pi_t, x_t} E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 \right) \right] \tag{5}$$

subject to (1).

Let $\Delta\tau$ be the (random) duration of the regime which started at time t. Then (5) can be

[15] It should be clear that we do not attempt to explain why α may be nonzero, i.e., why the central banks may commit for a finite number of periods.

[16] In the next section, we will consider the possibility of imperfect policy credibility, which we represent by a discrepancy between private sector's expectation of a regime change and the true α. Throughout the remaining of current section, we will assume that both the private sector and the central bank knows α and form expectations accordingly.

[17] As long as the interest rate does not appear in the loss function, (IS) equation is needed only to determine the required nominal interest rate to achieve the optimal values of the output gap and the inflation computed on step 1.

expressed recursively as

$$V_t = \min_{\pi_t, x_t} E_t \left(\sum_{i=0}^{\Delta\tau-1} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2 \right] + \beta^{\Delta\tau} V_{t+\Delta\tau} \right) \tag{6}$$

subject to (1)

where V_t is defined as a value function associated with the central banker's optimal loss at time t. This term appears because the central bank is assumed to take into account not only the losses accrued during her own regime but also the losses during all subsequent regimes. The latter is summarized by a terminal payoff $V_{t+\Delta\tau}$ in the objective function.

Since each central banker re-optimizes after a reappointment, at first sight this problem may be perceived as some sort of discretionary optimization. However, there is an important difference. Central bank's loss function involves a random running cost function (the first term on the right hand side). Rather than optimizing every period, the monetary authority commits to a policy rule that will last for random number of periods, which is (correctly) perceived by the private sector to have an average value of $\frac{1}{\alpha}$.[18] When her term ends unexpectedly, let's say, at $t + \Delta\tau$, her successor faces exactly the same type of problem. The recursive formulation implies that the solution to (6) will be optimal for the successive central bankers as well.

It is important to notice at this point that (6) contains only intra-regime expectations, i.e., expectations that are conditional on the commitment regime not changing. In other words, all the variables the policy maker has to forecast in (6) are determined within her regime. It will be shown later that, the policy maker whose term ends at time $t + \Delta\tau$ cannot affect the term $V_{t+\Delta\tau}$, since the value function at time $t + \Delta\tau$ will depend only on the realization of an exogenous shock, which obviously cannot be affected using the past information.[19] This feature—which is due to the forward looking nature of the model—will enable us to derive the policy rule analytically without appealing to iterative numerical methods.

3.2 Expectation Formation of the Private Agents

With probability α there will be a new regime, i.e., a re-optimization will take place; with probability $(1 - \alpha)$, current regime will continue, i.e., the policy maker will be applying the

[18] Note that the policymaker promises to continue the same rule as long as her term lasts.

[19] In general, when there are predetermined endogenous variables, this will not be possible. However, the expectations in (6) will still be determined within the regime.

rule she had committed at the beginning of her tenure. Accordingly, private agents form expectations that is conditional on different states, that is, private sector takes into account that the central banker can re-optimize next period. In other words, expectations can be decomposed into two terms: intra regime (within the regime) and inter regime (across the regimes). One period ahead inflation expectations will be formed as follows:

$$E_t[\pi_{t+1}] = \alpha E_t[\pi_{t+1}|inter\ regime] + (1-\alpha)E_t[\pi_{t+1}|intra\ regime]. \tag{7}$$

We will look for a linear solution in which the endogenous variables will be linear functions of the state. Since there are no predetermined endogenous variables in the model, state variables will be the Lagrange multiplier (will be denoted as φ_t) on the constraint (1), and the exogenous supply shock u_t.[20] Therefore, also exploiting the fact that systems dynamics are linear, it should be possible to express the expected inflation conditional on a regime as

$$E_t[\pi_{t+1}|inter\ regime] = cE_t[\varphi_{t+1}|inter\ regime] + dE_t[u_{t+1}|inter\ regime].$$

for some scalar constants c and d. Note that φ_{t+1} represents the central bank's promises regarding future policy. Recall that, upon a regime change, the policy maker re-optimizes, disregarding the past promises of previous central bankers. Thus, conditional on the re-optimization, the private agents' rational expectation of φ_{t+1} is zero. In notational terms

$$E_t[\varphi_{t+1}|inter\ regime] = 0.$$

On the other hand since the supply shock is i.i.d. with mean zero,[21] it follows that

$$E_t[\pi_{t+1}|inter\text{-}regime] = 0. \tag{8}$$

Hence, the only component of the private expectations that remains is the intra-regime expectations. Accordingly, constraint (1) can be simplified to

$$\pi_t = \kappa x_t + \beta(1-\alpha)E_t[\pi_{t+1}|intra\ regime] + u_t. \tag{9}$$

[20] See also Svensson and Woodford (2002)

[21] Suply shock is assumed to be i.i.d. for notational simplicity. Indeed, our results also apply to the more general case in which the supply shocks follows a more complex pattern.

3.3 Solving the Optimal Policy Rule

Given the linear quadratic nature of the current setup, the value function V will be a quadratic function of the state variables at the re-optimization period.[22] Consider a policy reformulation that occurs at time t. As argued above, at any period t, state variables will be the Lagrange multiplier φ_t denoting the value attached to satisfying past promises, and the exogenous shock u_t. On the other hand, φ_t will be zero at the period re-optimization since, by construction, the policymaker disregards past promises. Therefore, we postulate a value function such as

$$V = V(u). \tag{10}$$

where $V(u)$ is a (quadratic) function of the supply shock.

Notice once again that, due to the purely forward looking nature of the model, no state variables will survive across regimes, and thus, the new policymaker's constraints will be completely disconnected from the predecessors' acts.[23] In other words, the state is "reset" after each re-optimization. Therefore, current central banker's actions will have no effect on the successors' value function and thus the term $V(u)$ appears to be completely independent of the past actions. Since the problem of the policy maker will not depend on this term, it can be safely discarded from the objective function.

For notational simplicity, we will denote the 1-step ahead expectation of a variable X conditional on the regime not changing, that is $E_t[X_{t+1}|\text{intra regime}]$, by a special expectations operator $\tilde{E}_t[X_{t+1}]$. Accordingly, the central bank's problem can be restated as

$$\min_{\pi_{t+i}, x_{t+i}} E_t \left(\sum_{i=0}^{\infty} \beta^i (\lambda x_{t+i}^2 + \pi_{t+i}^2) \right) \tag{11}$$

subject to

$$\pi_t = \kappa x_t + \beta(1-\alpha)\tilde{E}_t \pi_{t+1} + u_t. \tag{12}$$

Incorporating the probability of a re-optimization into the minimization problem of the

[22] See Ljungqvist and Sargent (2000), Chapter (4).

[23] In the general case, when the structural model has endogenous state variables, current policy maker will internalize the loss of her successors.

policy maker, a central banker appointed at time t will face the Lagrangian formulation

$$E_t \sum_{j=1}^{\infty} (1-\alpha)^{j-1}\alpha \left(\beta^j V(u_{t+j}) + \sum_{i=0}^{j-1} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 \right. \right.$$
$$\left. \left. +\varphi_{t+i+1}(\pi_{t+i} - \kappa x_{t+i} - \beta(1-\alpha)\tilde{E}_{t+i}\pi_{t+i+1} - u_{t+i}) \right) \right) . \tag{13}$$

which can be simplified as

$$E_t \sum_{i=0}^{\infty} ((1-\alpha)\beta)^i \left(\alpha\beta V(u_{t+i}) + \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 \right) \right.$$
$$\left. +\varphi_{t+i+1}(\pi_{t+i} - \kappa x_{t+i} - \beta(1-\alpha)\tilde{E}_{t+i}\pi_{t+i+1} - u_{t+i}) \right) \tag{14}$$

where φ_{t+i+1}(predetermined at time $t+i$) is the Lagrange multiplier associated with the price setting equation at time $t+i$. The policy maker's goal is to choose a direct targeting rule in order to implement the output gap and inflation processes that minimize the Lagrangian. Taking derivatives with respect to π_t and x_t and rearranging gives

$$\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - x_{t+i-1}) \quad \text{for } i > 0 \tag{15a}$$
$$\pi_{t+i} = -\frac{\lambda}{\kappa}x_{t+i} \quad \text{for } i = 0 \tag{15b}$$

for all i belonging to the same regime.

At first sight, it may be puzzling to see that the policy rule is independent of the probability of a regime change, α.[24] However one has take into account that there is a stochastic process that leads to re-optimizations and thus expected rule at any time $t+i$ is a linear combination of the first order conditions (15) which can be written as

$$\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - (1-\alpha)x_{t+i-1}). \tag{16}$$

Two special cases are worth to mention. If $\alpha = 1$, i.e., when re-optimization occurs every period, these conditions reduce to the discretionary solution $\pi_{t+i} = -\frac{\lambda}{\kappa}x_{t+i}$ for all $t+i$. On the other hand, when $\alpha = 0$, i.e., current regime is expected to continue for an infinitely long period,[25] average condition (16) will be $\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - x_{t+i-1})$, which is identical to the

[24] Condition (15) also reflects the time inconsistent nature of the policy. The commitment chosen at any time t for time $t+1$, ceases to be optimal once the policy is reconsidered from time $t+1$ perspective.

[25] From the private sector's perspective this can be interpreted as perfect credibility—a situation in which the private agents believe that current policy rule will be followed forever.

full commitment under perfect credibility. Therefore, on one end of the spectrum, central bank has to re-optimize every period whereas on the other end, policy maker can credibly commit forever.

Note that (15) is the characterization of the optimal conditions that the central bank commits to fulfill as long its term lasts. On the other hand, private agents think that with probability α, another central banker takes office next period, adopting exactly the same policy from that period on. These expectations must be taken into account to compute the equilibrium and impulse responses.[26]

To gain more insight, the expected rule can be expressed as $x_{t+i} = -\frac{\kappa}{\lambda}\pi_{t+i} + (1-\alpha)x_{t+i-1}$. It is clear that optimal policy prescribes that current output gap should depend on its past realizations. This result, first stated by Woodford (1999), asserts that even though there is no intrinsic persistence in the structural model, optimal policy will involve "history dependence".[27] In other words, as long as there is a positive probability that current policy may last more than one period, the central bank behavior will involve inertia. Average degree of inertia varies inversely with α. The longer is the current regime expected to last, i.e, the higher is $1-\alpha$, the more inertial is the average optimal rule.[28]

Recall that we have defined the policy rule as a commitment to a condition involving the target variables, π_t and x_t. Condition (15) fits this definition. Accordingly, we will treat the optimality conditions as a direct targeting rule to which the policy maker commits during her term of office. Using the notation introduced above,

$$P_t(x_t, x_{t-1}, \pi_t) = x_t - x_{t-1} + \frac{\kappa}{\lambda}\pi_t \tag{17}$$

and thus,

$$\psi = [1, -1, \frac{\kappa}{\lambda}] \in \mathsf{R}^3. \tag{18}$$

3.4 Characterizing the Equilibrium

Optimality conditions (15) characterize the behavior of any central banker as long as she occupies the office. Obviously, upon a re-optimization, a new central banker will ensure

[26] See the section on impulse responses below.

[27] In the special case of α equal to 1 there is no history dependence since the optimal problem boils down to period by period optimization.

[28] Note that once the regime shock is realized, ex-post relationship between inflation and output gap may be different than the ex-ante average rule (16) would imply.

that the relation between inflation and output gap will be determined by (15a) rather than (15b). To characterize the equilibrium one has to take the stochastic policy reformulations into account—both inside and across regimes. Inside any regime, α will come into the picture through expectations. On the other hand, across regimes, α will affect the equilibrium through the realizations of regime shocks.

Accordingly, equilibrium will be computed in two steps. First, equilibrium processes will be solved within any regime. This can be achieved by substituting the optimality conditions in the aggregate supply equation and incorporating the possibility of a regime change into the expectations of private agents. Second, re-optimizations will be accounted by either averaging over the regime shocks or computing the equilibrium under a particular realization of regime shocks. We begin by defining the equilibrium that would have prevailed if no regime shocks were realized.

Definition 1 A within regime equilibrium is a rational expectations equilibrium in which $\{x_{t+i}, \pi_{t+i}\}$ satisfy the optimal policy rule (15) and the constraint (12) for all $i \geq 0$.

Interest rate is then set to implement the equilibrium in line with (2). Substituting (15) in (12), evolution of the output gap within any regime starting at period t will be determined by the following difference equation

$$(1-\alpha)\beta\tilde{E}_{t+i}x_{t+i+1} - (1 + \frac{\kappa^2}{\lambda} + (1-\alpha)\beta)x_{t+i} + x_{t+i-1} = \frac{\kappa}{\lambda}u_{t+i} \tag{19}$$

with an initial condition[29]

$$x_{t-1} = 0 \tag{20}$$

where, once again, we have used the linearity of the model to derive one period ahead output gap expectations. Notice that (19) contains only intra-regime expectations. We wish to solve (19) using the initial condition (20) and a transversality condition. The roots of the characteristic polynomial

$$(1-\alpha)\beta\mu^2 - (1 + \frac{\kappa^2}{\lambda} + (1-\alpha)\beta)\mu + 1 = 0 \tag{21}$$

are such that $0 < \mu_1 < 1 < \mu_2 < \infty$. Following Blanchard and Kahn (1980), (19) has a unique bounded solution consistent with the initial condition, and this bounded solution is the one

[29] This is a hypothetical initial condition to obtain the equilibrium. In practice x_{t-1} may or may not be zero. What is important here is to realize that at the period of a regime change, say at time t, the newly appointed monetary authority disregards the previous commitments, which in turn is equivalent to treating x_{t-1} as zero.

that satisfies transversality condition. For simplicity, nonetheless without loss of generality, let's assume the regime starts at time 0 so that initial condition reduces to $x_{-1} = 0$. Then, standard methods can be used to derive the within regime solution

$$x_t = \mu_1 x_{t-1} - a u_t \quad \text{for } \Delta\tau > t \geq 0 \tag{22}$$

where $a \equiv \frac{\kappa}{\lambda(1-\alpha)\beta\mu_2} = \frac{\kappa\mu_1}{\lambda}$ and μ_1 is a function of all the structural parameters and α. At first sight, the solution may look identical to the full commitment case. However, inertia in the equilibrium variables, μ_1, will be different—even within a specific regime—since agents contemplate that next period there may be a policy reformulation,[30] .

3.5 Impulse Response Functions

Equation (22) represents the equilibrium processes inside the regime starting at time zero and lasting for $\Delta\tau$ periods. Given that all the central bankers face exactly the same problem, one can characterize the impulse responses for the entire time domain. Note that there are two types of shocks in the model: structural shocks and regime shocks. There may be several ways to represent the impulse response functions depending on the nature of shocks and focus of attention:

i) Response to a supply shock averaging over the regime shocks (ex-ante response). This representation is particularly useful for comparing welfare under alternative levels of commitment. In this case impulse responses are given by[31]

$$\begin{aligned} E_t[x_{t+k}] - E_{t-1}[x_{t+k}] &= -a\mu_1^k(1-\alpha)^k u_t \\ E_t[\pi_t] - E_{t-1}[\pi_t] &= (1 - \tfrac{a\kappa}{1-\beta\mu_1(1-\alpha)})u_t \\ E_t[\pi_{t+k}] - E_{t-1}[\pi_{t+k}] &= -\tfrac{a\mu_1^k(1-\alpha)^k}{1-\beta\mu_1(1-\alpha)}u_t \text{ for } k \geq 1. \end{aligned} \tag{23}$$

Responses for π_t and x_t can be substituted into (3) to compute the expected optimal loss of the central bank.

ii) Response to a supply shock with no regime shocks. This represents a hypothetical case in which the current commitment regime lasts forever while all agents expect

[30] This can be chacked immediately by noticing that α appears in the characteristic equation 21.

[31] In the general case when there are endogenous state variables, computing the impulse responses is complicated (if not impossible), since one has to keep track of how many times and when a reoptimization occured between time t and $t + k$.

there may be a regime change with a fixed probability α. In this case the impulse responses are given by

$$E_t[x_{t+k}|\forall k>0:\nu_{t+k}=0]-E_{t-1}[x_{t+k}|\forall k>0:\nu_{t+k}=0]=-a\mu_1^k u_t$$
$$E_t[\pi_t]-E_{t-1}[\pi_t]=1-\tfrac{a\mu_1^k}{1-\beta(1-\alpha)\mu_1}u_t$$
$$E_t[\pi_{t+k}|\forall k>0:\nu_{t+k}=0]-E_{t-1}[\pi_{t+k}|\forall k>0:\nu_{t+k}=0]=\tfrac{a\mu_1^k}{1-\beta(1-\alpha)\mu_1}u_t$$

iii) Response to a supply shock given a draw of regime shocks (ex-post response). This representation provides a more realistic visualization of impulse responses as opposed to the average response in part (i). Suppose a sample series of regime shocks $\{\nu_{t+k}\}_{k=1}^{\infty}$ are realized in which $\nu_i=1$ (there is a re-optimization), if $i=\tau_0,\tau_1,\tau_2,...$ and $\nu_i=0$ (there is no re-optimization) otherwise. Using this notation, impulse responses will be given by

$$E_t[x_{t+k}|\{\nu_{t+k}\}_{k>0}]-E_{t-1}[x_{t+k}|\{\nu_{t+k}\}_{k>0}]=\begin{cases}0 & \text{if } \exists\ \tau_i \text{ s.t. } t<\tau_i\leq t+k\\ -a\mu_1^k u_t & \text{otherwise.}\end{cases} \tag{24}$$

and similarly for the inflation.

In what follows, we choose the first representation, that is, all the impulse response and welfare losses will be computed by averaging over regime shocks, since this case represents a more convenient framework to conduct relative welfare analysis. The calibration of the model parameters are mainly based on Woodford (2002). Benchmark values for the parameters are 0.99, 0.1 and 0.05 for β, λ, and κ respectively.[32] Each period is assumed to correspond to a quarter, while all variables are expressed annually.

Figure 1 shows the optimal welfare loss under varying degrees of commitment. In the first two panels, vertical axis represents optimum level of the loss. It is clear that, minimum loss that can be attained by the central bank diminishes as the average duration of commitments rises. The last panel of Figure 1 depicts, relative to pure discretion, the percentage gain accrued under varying degrees of commitment. Horizontal axis is the reciprocal of α, the expected duration in quarters of each regime. Interestingly, most of the gains from commitment is accrued at low degrees of commitment: an average regime length of 1 year is enough to cover 60% of the gains from commitment.

Dynamic responses to inflation and output gap to a temporary supply shock is depicted in Figure 2. The lines represented by $\alpha=0$ and $\alpha=1$ correspond to full commitment

[32] The main findings are quite robust to a wide range of deviations from the baseline calibration.

and pure discretion cases respectively, as in Woodford (1999). Under discretion, the output gap moves in sync with the shock, returning to its steady state once the shock is over. Under full commitment, the central bank behaves in an inertial way, promising mild output contractions, thereby influencing private sector expectations to its favor. The lines in between are the responses under imperfect commitment, with degrees of commitment .3 and .7. It is clear that, as commitments last longer, an optimizing central bank will act more inertial and the path of output gap and inflation will look qualitatively closer to full commitment.

4 Optimal Monetary Policy under Imperfect and Uncertain Credibility

In the previous section we were able to relax one of the assumptions underlying conventional optimal monetary policy under full commitment: central bank is allowed to commit for a finite duration. However, the policy maker's commitment—regardless of the duration of commitment—is still credible, in the sense that average duration of a commitment regime intended by the central bank coincides exactly with the private agents' expectations. This section attempts to explore how the optimal policy would alter when the central bank lacks some credibility of intention. Accordingly, in the rest of the study, we consider a set up in which the private agents' subjective view of the probability of a re-optimization is allowed to differ from that of the central bank's.

4.1 Representing Credibility

Referring to the stochastic re-optimization process that generates the policy re-optimizations in the previous setup, one may argue that α corresponds to a measure of credibility about the behavior of the central bank. Does this really represent imperfect policy credibility? In our view, the answer is no. The previous setup can be more easily interpreted as a situation in which the commitment is imperfect because of lack of ability—not because of lack of credibility. Therefore, the previous setup alone, is inadequate to capture the notion of credibility in the broad sense.

One way to represent imperfect credibility in our model, is to introduce the possibility that the private agents may expect commitment to last, on average, shorter than the true average duration intended by the policy maker. Denote the central bank's true average

frequency of re-optimization as α^o and the private sector's expectation of a regime change as α, so that imperfect credibility corresponds to the case $1 \geq \alpha > \alpha^o$.

Note that by definition, given α^o, the higher is α (the wider is the gap $\alpha - \alpha^o$), the lower is the credibility of the central bank. Normalizing with $1 - \alpha^o$ (the case with lowest possible credibility), one obtains a measure of the lack-of-credibility varying from 0 to 1. Finally, subtracting this value from one, we obtain $\frac{1-\alpha}{1-\alpha^o}$—a measure of credibility of intention. Since $1 \geq \bar{\alpha} \geq \alpha \geq \alpha^o \geq 0$, $1 \geq \frac{1-\alpha}{1-\alpha^o} \geq 0$.

When $\alpha = \alpha^o \neq 0$, the central bank operates under partial commitment of degree α^o with perfect credibility of intentions as in the previous section. The case $\alpha = \alpha^o = 0$ corresponds to full commitment under perfect credibility as in the conventional setups. If $\alpha > \alpha^o$, policy maker lacks some degree of credibility.

4.2 Representing Uncertain Credibility

In practice, central banks cannot directly observe private sector expectations, and thus, has to confront some degree of uncertainty. Motivated by this fact, we attempt to model imperfect credibility joint with uncertain credibility. To do this we introduce the following modification to the baseline model: Assume that at the beginning of each commitment regime, private sector's expectation of a regime change is drawn from a distribution with a support of $[\alpha^o, \bar{\alpha}]$. For simplicity, the lower bound of the support is assumed to be α^o, i.e., monetary authority is assumed to lack credibility only if the private sector's average expected regime duration is shorter than the central bank's actual intended regime duration.[33] Policy maker knows the true average frequency of re-optimizations, α^o, and the support $[\alpha^o, \bar{\alpha}]$, but not the exact private sector expectation, α. The distribution of α may be known or unknown to the central bank depending on the type of uncertainty one assumes.

In this set up, the central bank faces not only imperfect credibility ($\alpha^o \leq \alpha$), but also uncertainty about the extent of imperfection (α is unknown). Recall that we have constructed a measure of credibility in the form $\frac{1-\alpha}{1-\alpha^o}$. Given any support $[\alpha^o, \bar{\alpha}]$, uncertainty about α can be translated into uncertainty about the degree of credibility, as the central bank thinks her credibility of intention in the eye of the private agents may be any element in the set

[33] One can as well assume a more general case in which private agent's may expect both shorter and longer average durations than the one intended by the policy maker. However, this would complicate the analysis without providing any considerable extra insights.

$[0, \frac{1-\bar{\alpha}}{1-\alpha^o}]$. Note that, now, the model has two crucial parameters, namely α^o, and α. A nonzero α^o represents the finite nature of the commitment as in the previous section. On the other hand, whenever $\alpha > \alpha^o$, the monetary authority lacks some credibility of intention.

4.2.1 Types of Uncertainty

Since the seminal work of Brainard (1967), there has been a large number of research on monetary policy under model uncertainty. These studies can be categorized broadly under two main topics: Bayesian uncertainty, in which a prior distribution is imposed on some (or all) part of the model, or Knightian uncertainty, where the model perturbations are assumed to be in a bounded set, yet no unique prior distribution is imposed. Either the method is Knightian or Bayesian, main goal is the same: to explore implications of uncertainty for the conduct of monetary policy.

This study takes up both Knightian and Bayesian uncertainty. Accordingly, we solve for the optimal behavior of the central bank under two alternative types of uncertainty, both representing policy making under imperfect credibility. First, we consider a situation where the policy maker has no prior distribution over the uncertainty set. Next, we assume that the central bank knows the distribution of the private agents' expectations, α. In both cases the uncertainty is bounded. We begin our analysis with Knightian uncertainty.

4.3 Optimal Policy under Knightian Uncertainty

In this section, we adopt the Knightian approach to derive the optimal policy rule under imperfect credibility. This approach may provide useful insights in the sense that, in practice, uncertainty confronted by central banks is far too ambiguous to be represented by a unique prior distribution.

There are various approaches proposed in the literature to characterize Knightian uncertainty. Some authors (see e.g. Hansen and Sargent(2001)) have chosen a general approach in which uncertainty is introduced as a nonparametric perturbation to the model. However, nonparametric approaches for Knightian uncertainty provide little economic intuition about the specific nature of uncertainty the central banks may face in practice.

Some others have proposed solution methods for more structured uncertainty, such as uncertainty about the parameters of the model, or characteristics of certain shocks.[34]

[34] See, e.g. Giannoni(2002). Onatski and Stock (2000) provide a more structured representation of non

Here, our analysis is also structured by nature, since we seek to characterize intuitively a specific type of uncertainty: central bank's uncertainty about its own credibility. To our knowledge, this is the first attempt to analyze uncertain credibility in an optimal monetary policy framework.

4.3.1 Objective of the Central Bank Under Knightian Uncertainty

In this subsection, rather than imposing a prior distribution on the set of uncertainty, we assume that the central bank believes the private sector's expectation of a regime change, α, lies in a compact set, e.g., $\alpha \in [\alpha^o, \bar{\alpha}]$ and the exact distribution of α is not known, i.e., it can be anything including a given element holding with certainty.[35]

Moreover, suppose that the monetary authority is uncertainty averse in the sense axiomatized by Gilbao and Schmeidler(1989). In such a situation, the criterion adapted is a minmax approach: objective of the policy maker is to take precautions against the worst parameter configuration. In other words, the central bank acts "extremely cautious",[36] that is, the monetary authority tries to ensure a "reasonable" performance against the most unfortunate realizations of parameters in the uncertainty set by selecting the rule that minimizes the maximum loss. Consequently, decision makers solve a "minmax" problem in which agents implicitly play a zero sum game against a fictitious malevolent Nature. In our framework, this corresponds to the central bank choosing a policy rule while the Nature chooses an α.

It is worth to reemphasize the informational structure. The true frequency of re-ptimizations is α^o, which is known to the policy maker. However, at the beginning of each commitment regime, private agents' expectation of a regime change, α, is drawn from a distribution that is unknown to the central bank.[37] As in the previous section, the central bank faces a

parametric uncertainty.

[35] We rule out any learning process here. The central bank's uncertainty set stays intact through time. This is justified by the assumption, that, each time there is a regime change, private sector's beliefs are reset, and the duration of commitments are not long enough to learn about credibility. Our results go through as long as regime durations are finite.

[36] Caution in this sense means avoiding especially poor performances. In the monetary policy literature, a more common definition of caution is responding less aggressively to disturbances than the certainty case or aversion towards sharp movements in the policy instrument. See Martin and Salmon (1999) or Blinder (1998) for more discussion.

[37] It is common in the literature to assume uncertainty on behalf of the policy maker while the private agents know the true model. See Onatski and Stock (2000), Giannoni (2002) and Tetlow and Von Zur Muehlen (2000) for example. In our framework that would corresponds to the central bank's uncertainty about α^o while the private sector knows α^o. We do not pursue this approach in this study, since our focus is on imperfect

constraint such as

$$\pi_t = \kappa x_t + \beta(1-\alpha)\tilde{E}_t \pi_{t+1} + u_t, \tag{25}$$

except, in this case α is unknown. Each value of $\alpha \in [\alpha^o, \bar{\alpha}]$ corresponds to a different level of private sector expectation, i.e., a different degree of credibility.

At this point, we will introduce some notation that will be useful in the subsequent analysis. Given any policy rule ψ, and private sector expectations α, the equilibrium output gap and the inflation process can be computed by solving the system composed by the policy rule and the constraint (25). Denote the resulting equilibrium processes under any α and policy rule ψ as $x(\psi, \alpha)$ and $\pi(\psi, \alpha)$. It will be convenient to express the expected discounted loss under any ψ and α as $L[x(\psi, \alpha), \pi(\psi, \alpha)] = L(\psi, \alpha)$.

Moreover, given any α, it is possible to solve the optimal processes $x^*(\alpha)$ and $\pi^*(\alpha)$. Let $L^*(\alpha) = L[x^*(\alpha), \pi^*(\alpha)]$ denote the total expected discounted loss achieved at such an equilibrium. If we assume that optimal processes can be implemented by a policy rule $\psi^*(\alpha)$, it is then possible to write $L(\psi^*(\alpha), \alpha) = L^*(\alpha)$ at the global optimum.[38]

Using the notation just laid out, objective of the robust central bank is to choose the following policy rule.

Definition 2 A robust optimal policy rule is a specific targeting rule in which the policy maker solves

$$\min_{\psi \in \Psi} \left\{ \max_{\alpha \in [\alpha^o, \bar{\alpha}]} L(\psi, \alpha) \right\}. \tag{26}$$

Following Giordani and Söderlind (2002) the minmax problem implicitly assumes that the Nature maximizes when and only when the central bank optimizes. The interpretation is that, every time the central bank re-optimizes, she will have to deal with uncertainty, and design a robust rule. This approach is consistent with the perfect commitment solutions of the robust optimal monetary policy in the literature.[39]

and uncertain credibility.

[38] Obviously, magnitude of the loss function also depends on other parameters of the structural equations. However, these parameters are assumed to be known to both parties, and thus will be treated as fixed.

[39] See, for example, the applications in Hansen and Sargent (2001).

4.3.2 Solution Method

The minmax problem (26) can be solved by using the property of a hypothetical zero sum game between policy maker and a malevolent nature.[40] The central bank's objective is to choose a policy rule so as to minimize the welfare loss given that Nature tries to hurt her as much as possible. On the other hand, Nature chooses an α to maximize the loss $L(\psi, \alpha)$ knowing that the policy maker will minimize it. The idea is to find a pair of (ψ, α) that are best responses to each other, and thus that constitute a Nash equilibrium of the zero sum game. But in such a game, the equilibrium action of each player will be the solution to the minmax problem for both parties.[41] By choosing a robust optimal policy rule, the central bank can guarantee that the loss will not be higher than the following minmax equilibrium.

Definition 3 A minmax equilibrium is a bounded rational expectations equilibrium $x^*(\psi^*, \alpha^*)$ and $\pi^*(\psi^*, \alpha^*)$, where $\psi^* \in \Psi$ is a robust optimal policy rule and α^* maximizes the loss $L(\psi^*, \alpha)$ on the constraint set $[\alpha^o, \bar{\alpha}]$.

The solution method based on Giannoni (2002), can be summarized as follows:

1.Find the optimal equilibrium processes $x^*(\alpha)$ and $\pi^*(\alpha)$ for any $\alpha \in [\alpha^o, \bar{\alpha}]$.

2.Compute the loss function $L^*(x^*(\alpha), \pi^*(\alpha)) = L^*(\alpha)$. Calculate the candidate worst case parameter, i.e., find the solution to the following problem

$$\alpha^* = \arg \max_{\alpha \in [\alpha^o, \bar{\alpha}]} L^*(\alpha).$$

The process $\{x^*(\alpha^*), \pi^*(\alpha^*)\}$ is the candidate minmax equilibrium.

3.Find the optimal policy rule ψ^* that implements the minmax equilibrium in step 2.

4.Verify that (ψ^*, α^*) is a global Nash equilibrium, i.e., that there is no $\tilde{\alpha} \in [\alpha^o, \bar{\alpha}]$ satisfying

$$L(\psi^*, \alpha^*) < L(\psi^*, \tilde{\alpha}) \tag{27}$$

Once a global Nash equilibrium is attained, the solution method guarantees that ψ^* is the robust policy rule we are looking for.

[40] See Giannoni (2002) for a detailed exposition.

[41] See Osborne and Rubinstein (1994) for a proof.

4.3.3 Solving the Optimal Rule under Imperfect Credibility

In this part, equilibrium processes will be derived for any level of credibility. That means we will compute the optimal policy rule under the assumption that the policy maker faces the constraint (25) with a fixed α. Next, the subsequent part will determine the minmax equilibrium, and the robust optimal policy rule—the policy rule that performs "reasonably well" across a range of private sector expectations $\alpha \in [\alpha^o, \bar{\alpha}]$ or equivalently, a range of credibility $[0, \frac{1-\bar{\alpha}}{1-\alpha^o}]$ that implements it. We will seek for a policy rule of the form (18).

Consider the central bank's problem at the beginning of a regime starting at period t. Lagrangian of the policy maker for a given level of credibility, can be formulated as

$$E_t \sum_{j=1}^{\infty} (1-\alpha^o)^{j-1}\alpha^o \left(\beta^j V(u_{t+j}) + \sum_{i=0}^{j-1} \beta^i \left(\lambda x_{t+i}^2 + \pi_{t+i}^2 + \varphi_{t+i+1}(\pi_{t+i} - \kappa x_{t+i} - \beta(1-\alpha)\tilde{E}_{t+i}\pi_{t+i+1} - u_{t+i}) \right) \right). \tag{28}$$

Switching over sums, (28) can be simplified as

$$E_t \sum_{i=0}^{\infty} ((1-\alpha^o)\beta)^i \left(\alpha^o \beta V(u_{t+i}) + \left((\lambda x_{t+i}^2 + \pi_{t+i}^2) + \varphi_{t+i+1}(\pi_{t+i} - \kappa x_{t+i} - \beta(1-\alpha)\tilde{E}_{t+i}\pi_{t+i+1} - u_{t+i}) \right) \right). \tag{29}$$

Taking derivatives with respect to π_{t+i} and x_{t+i} and rearranging, optimality conditions can be stated as

$$\pi_{t+i} = -\frac{\lambda}{\kappa} x_{t+i} \quad \text{for } i = 0 \tag{30a}$$

$$\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - \frac{1-\alpha}{1-\alpha^o} x_{t+i-1}) \text{ for } i > 0. \tag{30b}$$

Note, once again that these conditions have to be satisfied inside the regime which started at time t. Since all the policy makers face the same problem, equation (30) can be generalized for any regime starting at time τ, simply by replacing t with τ.

We have just computed the solution under the assumption that private sector's expectation of a regime change is α. Recall that, credibility is defined as the ratio of the private sector's and central bank's subjective probabilities of regime staying the same, namely, $\frac{1-\alpha}{1-\alpha^o}$. Note that, in the first period of the commitment regime, optimal commitment rule does not

depend on the degree of credibility. However after the first period, more credible central bank will find it optimal to commit to a more history dependent rule, since condition (30b) states that, the monetary authority's optimal behavior depends on the perception of her own credibility in the eye of the private agents.

The next step of the solution is to compute the candidate worst case parameter. To do this, first, optimal equilibrium processes $x^*(\alpha)$ and $\pi^*(\alpha)$ are computed by solving the system composed of the constraint (25) and the optimality conditions (30a) and (30b). This is equivalent to solving the difference equation

$$(1-\alpha)\beta\tilde{E}_{t+i}x_{t+i+1} - (1 + \frac{\kappa^2}{\lambda} + \frac{(1-\alpha)^2}{1-\alpha^o}\beta)x_{t+i} + \frac{1-\alpha}{1-\alpha^o}x_{t+i-1} = \frac{\kappa}{\lambda}u_{t+i}$$

which is obtained by plugging the first order conditions into equation (25) together with an initial condition $x_{t+i-1} = 0$. The solution turns out to be in the same form as (22) with different a and μ_1 parameters.

Next, optimal welfare loss is derived by substituting relevant variables into the loss function (3). Formally, we compute $L(x^*(\alpha), \pi^*(\alpha)) = L^*(\alpha)$. The resulting expected loss as a function of α, for $\alpha^o = 0$ for example, is given by

$$L^*(\alpha) = \left(\frac{\kappa a(\alpha)}{1-(1-\alpha)\beta\mu_1(\alpha)}\right)^2 + \lambda a^2 \left(\frac{\kappa}{1-\beta\mu_1^2(\alpha)} + 1 - \frac{2\kappa a(\alpha)}{1-(1-\alpha)\beta\mu_1(\alpha)}\right) u^2.^{42} \quad (31)$$

Therefore, as postulated in section 2, optimal loss is a quadratic function of only the supply shock u.[43]

To find the candidate worst case parameter α^*, i.e., the private sector expectations that maximizes (31), Figure 3 illustrates the value of the optimal loss function $L^*(\alpha)$ on the entire domain for varying levels α^o. Recall that, the lower bound of private sector's expectation of a regime change is, α^o.[44] For any given value of $\bar{\alpha}$ (taken as 1 to cover the most general case), the domain $[\alpha^o, \bar{\alpha}]$ becomes smaller as α^o is higher. The line that corresponds to $\alpha^o = .8$, for example, is drawn on the domain $[.8, 1]$ while the line that represents $\alpha^o = 0$ shows the loss across the domain $[0, 1]$. It is clear that, regardless of the natural duration of a regime,

[42] Although μ_1 and a are nonlinear functions of many parameters of the model, they are represented as a function of α since all the structural parameters except for α are assumed to be known.

[43] Expected discounted loss is computed by averaging over regime shocks.

[44] Note that in this case, the policy maker is able to commit for $\frac{1}{\alpha^o}$ periods, whereas the private sector thinks that the commitment will last $\frac{1}{\alpha}$ periods which is less or equal to $\frac{1}{\alpha^o}$ since $\alpha \in [\alpha^o, 1]$.

the total loss is strictly increasing in $[\alpha^o, 1]$, implying a corner solution to the maximization problem in step 2, that is,

$$\alpha^* \equiv \arg \max_{\alpha \in [\alpha^o, \bar{\alpha}]} L^*(\alpha) = \bar{\alpha}. \tag{32}$$

Given the forward looking setting, this result is intuitive. A higher α represents a lower credibility level. The higher the α, the more the private agents believe that the central bank is likely to renege on the future promises. Therefore, higher α will weaken the central bank's ability to bring down any inflationary pressure by promising future contractions in output. This will, in turn, lead to less favorable trade-off between output and inflation, and consequently, higher welfare loss will be realized with regard to the high credibility case. Therefore, central bank's loss will be maximized when $\alpha = \bar{\alpha}$.

4.3.4 Optimal Robust Rule

The next step is to find a policy rule that will implement the minmax equilibrium. Robust optimal policy rule is characterized by simply finding the central bank's best response against $\bar{\alpha}$ which is equivalent to solving the optimization problem with a constraint given by

$$\pi_t = \kappa x_t + \beta(1 - \bar{\alpha})\tilde{E}_t \pi_{t+1} + u_t. \tag{33}$$

It is straightforward to see that resulting policy is exactly of the same form as (30) except α is replaced by $\bar{\alpha}$. Therefore, the robust policy rule for any central bank re-optimizing at period t can be characterized as

$$\pi_{t+i} = -\frac{\lambda}{\kappa} x_{t+i} \qquad \text{for } i = 0 \tag{34a}$$

$$\pi_{t+i} = -\frac{\lambda}{\kappa}\left(x_{t+i} - \frac{(1-\bar{\alpha})}{1-\alpha^o} x_{t+i-1}\right) \text{ for } i > 0. \tag{34b}$$

Noting that $\alpha^o \leq \bar{\alpha}$, (34) reveals an interesting result: optimal robust policy rule under uncertain credibility ($\alpha^o < \bar{\alpha}$) will involve less history dependence than the optimal rule under perfect credibility ($\alpha^o = \bar{\alpha}$). The higher is the upper bound of the uncertainty set, the closer is the robust optimal policy to the pure discretionary case. In the extreme case $\alpha = 1$, i.e., when the policy maker thinks she may not have credibility at all, robust optimal policy rule exactly coincides with the discretionary solution. Therefore, one can conclude that under uncertain credibility, robust optimal policy will look observationally closer to the discretionary policy.

It is important to realize that imperfect and uncertain credibility are interrelated concepts. One may construct a central bank credibility measure by using the size of the uncertainty set about credibility, since low credibility will most likely be associated with high uncertainty about credibility. Therefore, a more general credibility measure would be $\frac{1-\bar{\alpha}}{1-\alpha^0}$, which is exactly the term on the past output gap in the robust targeting rule (34b). Once again, we conclude that lower credibility calls for a less history dependent rule. The lower is the credibility the closer is the central bank's behavior to discretionary solution. Thus, imperfect policy credibility may be one of the reasons why the observed behavior of the central banks deviate from full commitment behavior, in the sense that interest rates seem to be far less inertial than a fully commitment regime would prescribe.

4.3.5 Verifying the Minmax Equilibrium

The final step is to check if the solution constitutes a global Nash equilibrium, i.e., if α^*, ψ^* are globally best responses to each other. This is equivalent to verifying that

$$\arg \max_{\alpha \in [\alpha^o, \bar{\alpha}]} L(\psi^*, \alpha) = \bar{\alpha}. \tag{35}$$

Condition (35) can be verified by computing the loss under the candidate policy rule ψ^* for any $\alpha \in [0, 1]$ and checking if the corresponding loss function attains a maximum at α^*. Under the policy rule ψ^*, the law of motion of equilibrium variables $x(\psi^*, \alpha)$ and $\pi(\psi^*, \alpha)$ inside any regime starting at time t will be determined by the difference equation[45]

$$\beta(1-\alpha)\tilde{E}_{t+i}x_{t+i+1} - (1 + \frac{\kappa^2}{\lambda} + \beta(1-\alpha)(1-\bar{\alpha}))x_{t+i} + (1-\bar{\alpha})x_{t+i-1} = \frac{\kappa}{\lambda}u_{t+i}.$$

with an initial condition of

$$x_{t-1} = 0.$$

It follows that, equilibrium processes take the same form as in the previous section with different parameters of a and μ_1. Figure 4 depicts the resulting $L(\psi^*, \alpha)$ for $\alpha \in [0, \bar{\alpha}]$ and $\bar{\alpha}$ (or equivalently α^*) takes values from .2 to .8 in increments of .2. Each α^* corresponds to a different robust rule. For example, when $\alpha^* = .2$, the policy maker behaves as if the private agents expect the regime to last 5 quarters on average. Clearly, the loss function is monotonically increasing in α and thus Nature will still choose the highest α, namely $\bar{\alpha}$, to

[45] Without loss of generality, α^o is fixed to 0.

hurt the policy maker as much as possible. The result remains intact for any $\alpha^o \in [0,1]$. Consequently, $(\alpha^*;\psi^*) \equiv (1, -\frac{(1-\bar{\alpha})}{1-\alpha^o}, \frac{\kappa}{\lambda}; \bar{\alpha})$ constitute a global Nash equilibrium and (34) is the robust optimal rule.

4.3.6 Robust Rule Versus Baseline Rule

Performance of the robust rule under uncertainty can be contrasted with the optimal imperfect commitment rule under known credibility. This is demonstrated in Figure 5 with an uncertainty set $[0.1, 0.5]$ and a true value of $\alpha = 0.1$. In this example, the central bank is able to commit to rule for 10 quarters on average, however she thinks that due to lack of credibility, private sector may expect the commitment to end after as low as 2 quarters. If the central bank is a robust decision maker, it turns out that she takes action as if the private agents expect the commitment to last 2 quarters. This behavior smooths the central bank's risk as shown in Figure 5. Hence, if the credibility is actually low, the central bank, by behaving robustly, achieves a better outcome (lower welfare loss) than she would have achieved had she committed to a baseline rule. However, if the central bank's pessimism turns out to be wrong, the baseline rule delivers much better results.

One last remark is in order. The robust equilibrium is a hypothetical, yet useful concept, created to obtain the robust behavior of the agents. On the other hand, the realized equilibrium, in general, is distinct from the robust equilibrium. For example, the robust equilibrium impulse response of the output gap to a supply shock averaged over regime shocks is given by

$$E_t[x_{t+k}] - E_{t-1}[x_{t+k}] = -a\mu_r^k(1-\bar{\alpha})^k u_t, \tag{36}$$

while the realized response would be

$$E_t[x_{t+k}] - E_{t-1}[x_{t+k}] = -a\mu_a^k(1-\alpha^o)^k u_t. \tag{37}$$

where μ_r and μ_a are not necessarily same.

4.4 Credibility and Bayesian Uncertainty

Suppose natural frequency of regime shocks, α^o, is known to the central bank, but at the beginning of each regime, the private agents' beliefs are drawn from a uniformly distributed process over $[\alpha^0, \bar{\alpha}] \subset [0,1]$. Once again, if we look for an equilibrium in which system dynamics are linear, the Lagrangian of the central bank optimizing at time t can be formulated

as

$$E_t \sum_{i=0}^{\infty} ((1-\alpha^o)\beta)^i \Big(\alpha^o \beta V(u_{t+i}) + \Big[(\lambda x_{t+i}^2 + \pi_{t+i}^2) + \varphi_{t+i+1}(\pi_{t+i} - \kappa x_{t+i} - \beta(1 - \frac{(\alpha^0 + \bar{\alpha})}{2})\tilde{E}_{t+i}\pi_{t+i+1} - u_{t+i}) \Big] \Big) \tag{38}$$

Taking derivatives with respect to π_t and x_t, and substituting out the Lagrangian multiplier, we obtain the following optimality conditions that has to be satisfied within the regime starting at period t

$$\pi_{t+i} = -\frac{\lambda}{\kappa} x_{t+i} \quad \text{for } i > 0,$$

$$\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - \frac{1}{2}(1 + \frac{1-\bar{\alpha}}{1-\alpha^o})x_{t+i-1}) \text{ for } i > 0. \tag{39}$$

Recall that, one of the measures of credibility constructed in the paper is $\frac{1-\bar{\alpha}}{1-\alpha^o}$. Equation (39) reveals that the higher is the credibility, the more weight is attached to past output gap and thus, the closer is the policy rule to full commitment behavior. Under perfect credibility, i.e., when $\bar{\alpha} = \alpha^o$, policy replicates the full commitment solution. In all other cases, monetary policy is biased towards discretion.

The policy rule (39) represents the central bank's behavior within a specific regime. There will be a re-optimization with probability α^o, upon which the subsequent central banker will choose exactly the same state contingent rule. The policy rule averaged over true regime shocks, will be given by

$$\pi_{t+i} = -\frac{\lambda}{\kappa}(x_{t+i} - (1 - \frac{1}{2}(\bar{\alpha} + \alpha^o)x_{t+i-1}). \tag{40}$$

Once again, when $\alpha^o = 1$, i.e., if the central bank has to re-optimize every period, optimal policy reduces to the discretionary behavior. On the other hand, the case $\bar{\alpha} = \alpha^o > 0$ corresponds to the solution under imperfect commitment with perfect credibility of intentions.

5 Summary and Conclusions

The main goal of this paper has been to present an analytical and intuitive way of representing the central bank behavior from a discretion versus commitment standpoint. Using a baseline forward looking model, two basic setup has been proposed to allow deviation from the full commitment case. Namely, we have incorporated imperfect credibility, and finite lasting

commitments into the baseline model. This allowed us to introduce the concept of imperfect commitment.

Finite regime duration is of particular interest from a practical point of view, since no central bank—even under perfect credibility—can commit for infinite number of periods, due to presence of unavoidable factors such as reappointment of the central bank administration, "large" shocks, institutional changes, etc.... Hence, even if the individual policy maker has perfect credibility of intentions, it is possible to talk about an imperfection in the commitment process due to frictions just mentioned. To incorporate this issue into the optimal monetary policy framework, we have constructed a setup in which the central bank can reformulate the policy with an average frequency that is known to the private agents. In other words, the private agents truly contemplate policy intentions. This setup allowed us to obtain a unique rational expectations equilibrium, without dealing with reputational problems. Consequently, we have characterized the equilibrium under a continuum where full commitment and discretion corresponds to two edges of the spectrum. Most of the gains from commitment is accrued under low degrees of commitment, confirming the central banks' concern about achieving a stable level of credibility. In other words, as long as the monetary authority's policy intentions match to the private expectations, even a commitment lasting a couple of quarters is enough to cover most of the gains from commitment.

As a second step to model the deviations from the full commitment behavior, we relaxed the assumption of perfect credibility of intentions by allowing the private agents to expect the commitment regime to end, in general, sooner than that is intended by the policymaker. Moreover, we also incorporated the case of uncertain credibility into the analysis by letting the private sector's beliefs about the future course of monetary policy to be drawn from an unknown distribution. Accordingly, we solve the robust optimal monetary policy under uncertain credibility. Central bank's optimal response to higher uncertainty and lower credibility is to act closer to the discretion, pursuing a less history dependent rule.

To summarize, in this paper, observed deviations of policy behavior from "frictionless" case of full commitment is decomposed into two sources: finite commitment and lack of credibility. Under finite commitment, individual policy behavior within any regime looks identical to full commitment, whereas the overall policy across regimes is disrupted by re-optimizations. Under imperfect credibility, on the other hand, policy changes behaviorally—even within a specific regime. Moreover, robust optimal policy prescribes a less history

dependent policy under uncertain credibility.

An extension for future research would be to allow the central bank's uncertainty set to shrink through time, by introducing a learning process. This may provide insights to understand the adjustment of the economy during a stabilization regime.

If the Fed behavior after 1980's had become more credible—as claimed by many scholars and practitioners of monetary policy—then our model would suggest higher inertia after 1980's. An interesting question at this point is whether imperfect commitment proposed in this study can account for the differences between the theoretical and empirically observed instrument rules in the recent literature, which will exactly be the focus of a companion study to this paper.

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Figure 1: Optimal Welfare Loss with Semi-commitment

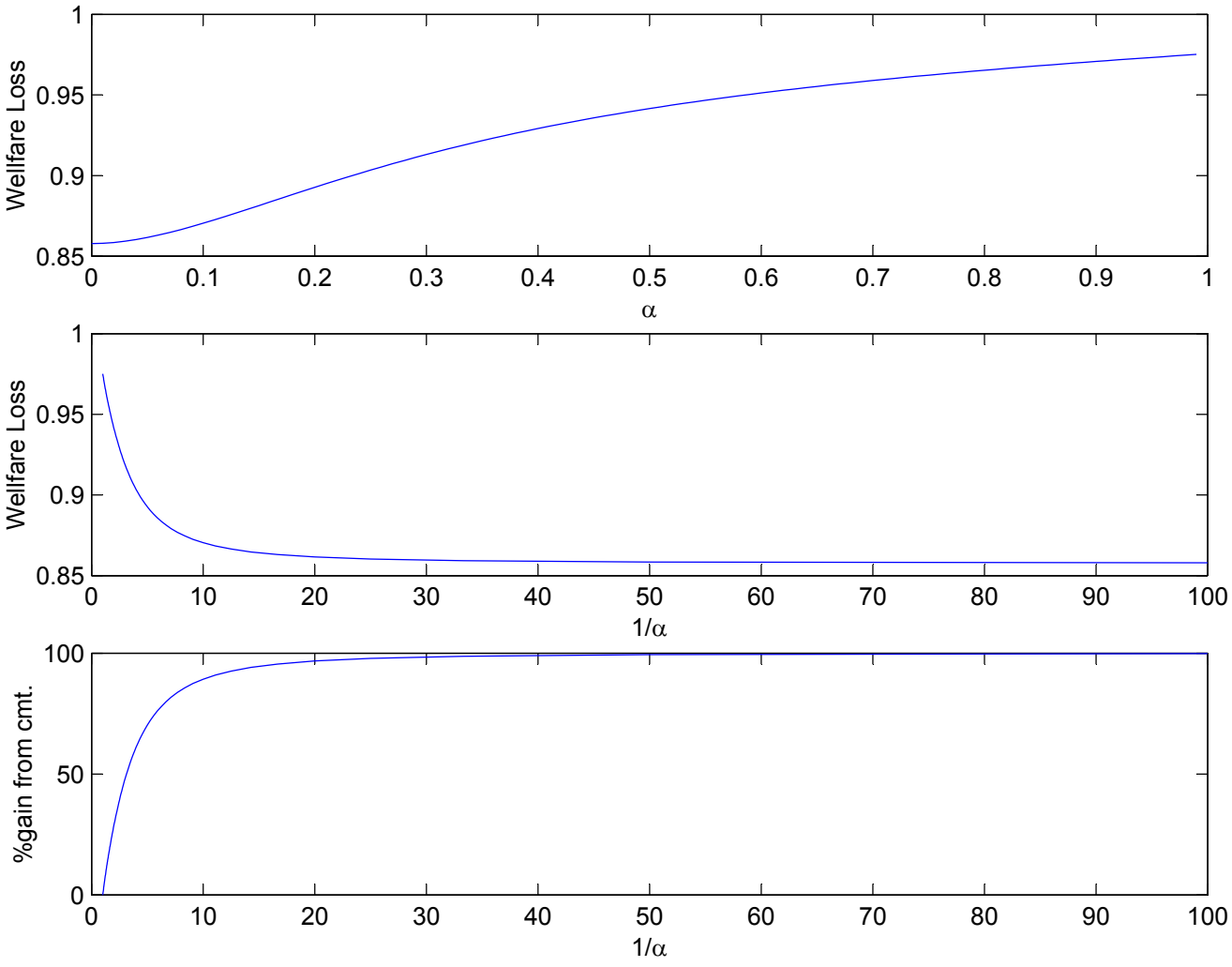

Figure 2: Impulse responses to inefficient supply shock

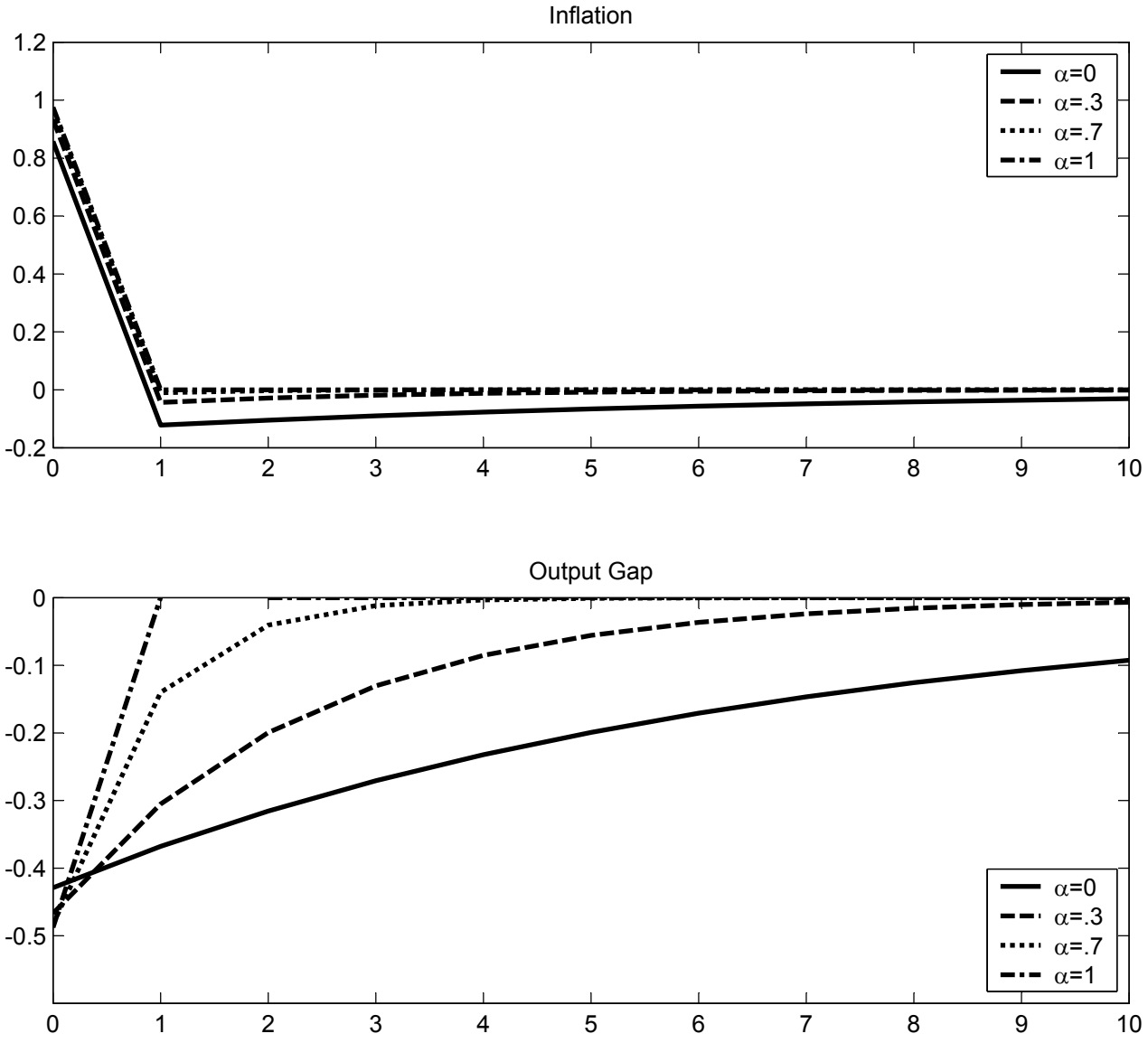

Figure 3: Welfare Loss under Minmax Equilibrium

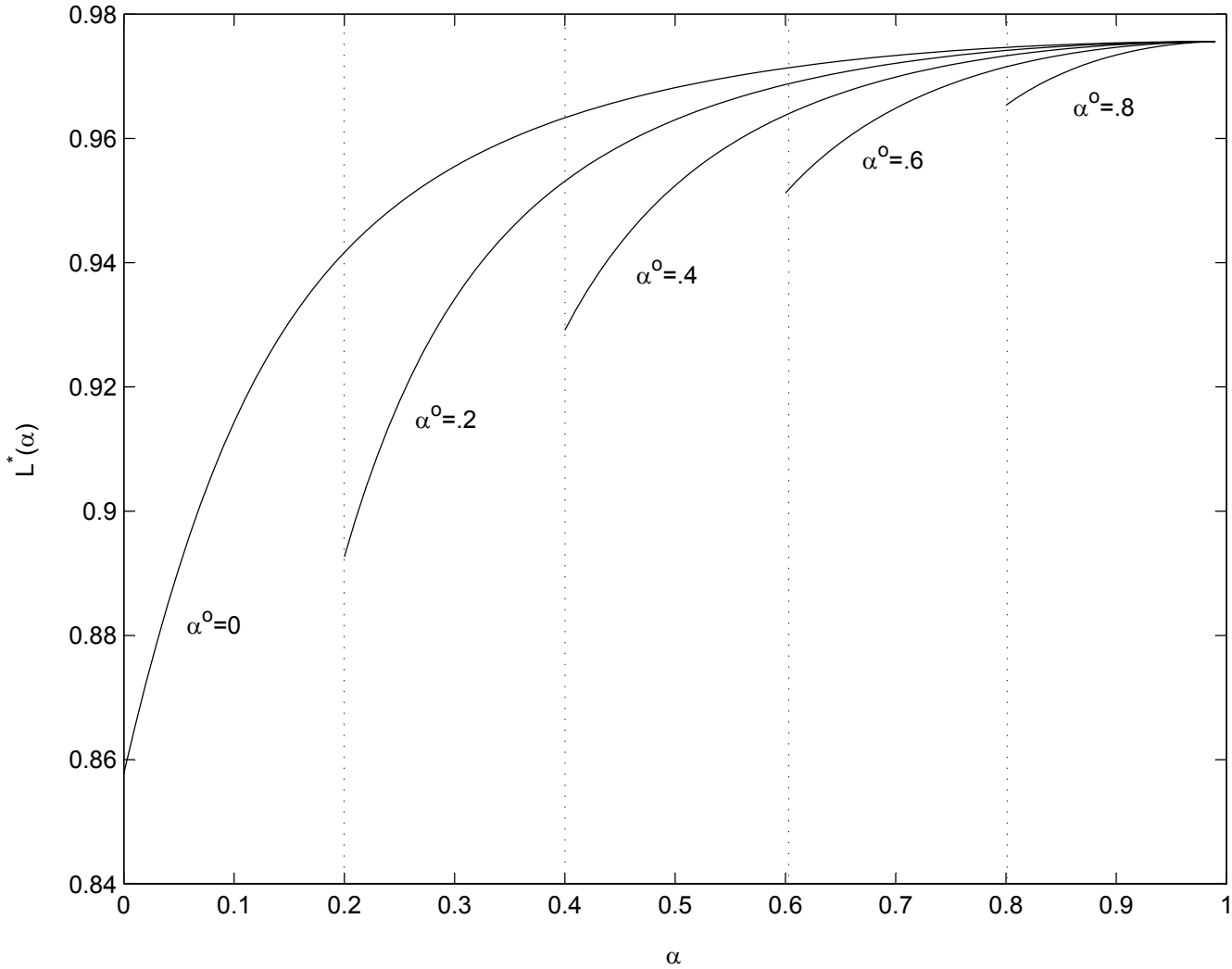

Figure 4: Welfare Loss Under Various Minmax Rules

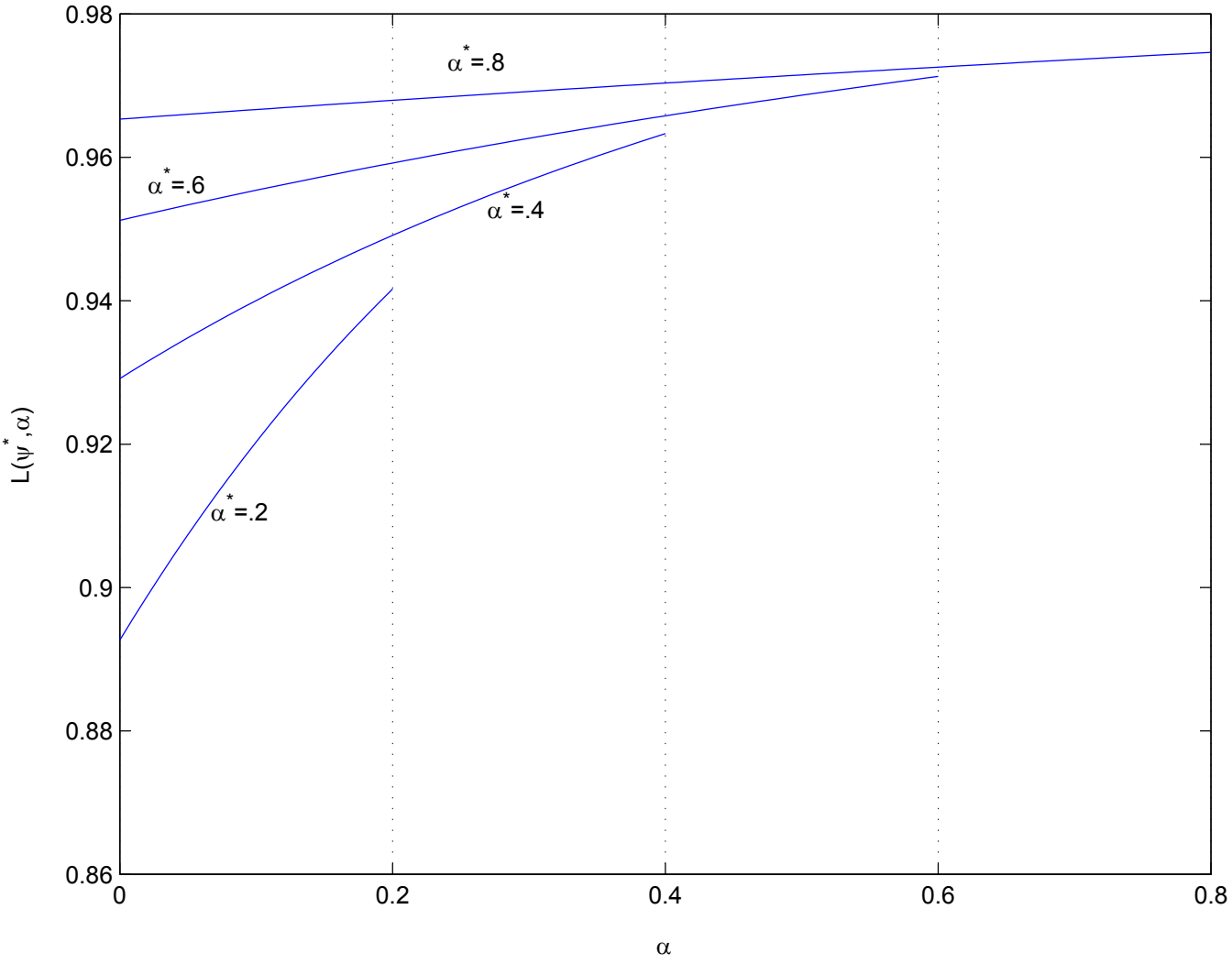

Figure 5: Welfare Loss under Robust Rule and Baseline Rule ($\alpha \in [.1,.5]$)