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# Nonlinearity and Structural Change in Interest Rate Reaction Functions for the US, UK and Germany

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## Abstract

This paper analyses monthly values of the short-term interest rate for the US, the UK and Germany since the early 1980s in the context of possible nonlinearities and changes over time in the interest rate response to the output gap, inflation, past interest rate changes and external variables (world commodity prices and the real exchange rate). The statistical models used are of the smooth transition class, with very substantial evidence of nonlinearity and/or parameter instability uncovered in the interest rate reaction functions for all three countries. These effects are primarily associated with time and changes in interest rates, with different coefficients applying when interest rates are increasing versus when they are decreasing. The reaction function coefficients for both the US and UK are also found to change during the 1980s.

**JEL Classification**: C51, E52, E58.

**Keywords**: Monetary policy, Nonlinear models, Structural change, Smooth transition models

## 1. Introduction

There is a huge literature concerning interest rate reaction functions. These studies are often expressed in terms of the so-called Taylor rule, which captures the interest rate response of the monetary authority to inflation and real output (or the output gap). However, almost all of this literature assumes that these interest rate responses are linear and time-invariant. In terms of their theoretical underpinnings, these linear models are based on two key assumptions, namely, a linear Phillips curve and a quadratic loss function for the preferences of the central bank (see, for instance, Clark *et al,* 1999, Clarida and Gertler, 1997, Clarida *et al,* 1998, 2000, Gerlach and Schnabel, 2000). Further, in imposing time-invariant reaction functions, the parameters of the Phillips curve and the loss function are assumed to be constant over time.

A number of theoretical and empirical studies in the very recent literature have questioned the two assumptions underpinning linearity. For example, Schaling (1999) and Dolado, María-Dolores and Naveria (2004) examine the implications of a nonlinear Phillips curve, while Nobay and Peel (2003) and Ruge-Murciá (2002, 2004), among others, challenge the assumption of a quadratic loss function. Other studies that find empirical support for the presence of nonlinearity in central bank interest rate reaction functions include Kim, Osborn and Sensier (2004), Martin and Milas (2004) and Bec, Salem and Collard (2002).

The nonlinear interest rate reaction functions estimated to date assume that the nonlinearity is related to the value of the output gap and/or the inflation deviation from target (for example, Bec *et al*. 2002, Dolado *et al*., 2004) with the parameters of the models assumed to be otherwise time-invariant. Nevertheless, in the context of interest rate reaction functions it is widely acknowledged that the actions of central banks have changed over the postwar period. This is, perhaps, most evident in the context of the US Federal Reserve Board, where a number of studies allow the coefficients of the reaction function to change with the chairman of the Federal Reserve, with distinct reaction functions sometimes estimated for the tenure of each Fed Chairman; see, in particular, Judd and Rudebusch (1998). Nevertheless, even when structural change is permitted, constancy of US monetary policy is almost invariably assumed from around 1983 onwards, that is, after the end of the atypical period when the Fed targeted nonborrowed reserves (1979-1982). In contrast, structural change may be expected to be prevalent for the UK, which has experienced a number of changes in monetary policy in the period after 1970 and where inflation targeting was adopted only in 1992 (Nelson, 2000). Clarida, Gali and Gertler (1998) assume a structural break in late 1990 for the UK, but treat German monetary policy as essentially constant from 1979.

The present paper examines the possibility of both nonlinearity and structural change in the interest rate reaction functions of the US, the UK and Germany since the early 1980s. Our framework is the class of smooth transition regression (STR) models. This class is particularly attractive here, since it allows monetary policy to evolve over time. In other words, the "structural breaks" considered can be relatively smooth, rather than necessarily abrupt. Lundbergh, Teräsvirta and van Dijk (2003) discuss a general specification of a STR model that encompasses both nonlinearity and structural change, which permits us to examine nonlinearity in monetary policy while also considering possible parameter evolution over time. Our analysis begins in 1984 in order to abstract from the period of high interest rates in the 1970s and early 1980s. For the US, this implies we specifically exclude the subperiod of nonborrowed reserved targeting and consider only the period under Alan Greenspan's chairmanship of the Federal Reserve Board where monetary policy is typically assumed to be time invariant.

The structure of this paper is as follows. Section 2 outlines the specification we adopt for the interest rate reaction function, together with the data used. Our substantive results are then presented in Section 3, with concluding comments in Section 4. Further details of our STR modelling methodology can be found in the Appendix.

## 2. Interest Rate Models

### 2.1 The Models

For the central banks, the main operating instrument of monetary policy is a short-term interest rate, which is usually an interbank lending rate for overnight loans. Therefore, an empirical reaction function describes how the central bank sets this short-term interest rate, and in doing cares about stabilizing inflation and output.

In its usual linear form the interest rate reaction function can be expressed as

$$r_t = \alpha' w_t + u_t \qquad (1)$$

where $r_t$ is the short-term interest rate, $w_t$ is $(p \times 1)$ vector of explanatory variables, typically including a constant, $\alpha$ is a $(p \times 1)$ coefficient vector, while $u_t$ is assumed to be $i.i.d.(0,\sigma^2)$. The literature following Taylor (1993)[1] assumes that the central bank adjusts the nominal short-term interest rate in response to the (past or forecast) gaps between inflation and output in relation to their targets. Typically, lagged values of the interest rate are also included in (1) to capture dynamics, often expressed as interest rate smoothing by the central bank (Clarida *et al*. 2000).

Since we wish to make no assumptions about the source of any nonlinearity in interest rate reactions and also to avoid simultaneity problems, our models are of the reduced form type, so that we use past values[2] for the output gap and inflation in (1). However, to reflect other variables examined by the central bank and following many previous studies, we also allow world commodity price inflation to enter the reaction function for all three countries. Further, Clarida *et* al. (1998) find the real exchange rate to be important for German monetary policy; we allow this variable to play a role for both Germany and the UK, since these are open economies.

Although we present linear models based on (1), our primary interest is in the interest rate reaction functions specified and estimated using the STR methodology to allow for nonlinearity and/or structural change. The models of this type presented below can be written as

$$r_t = \beta_0' w_t + \beta_1' w_t\, F_1(s_{1t}) + \beta_2' w_t\, F_2(s_{2t}) + u_t \qquad (2)$$

where $s_{1t}$, $s_{2t}$ are distinct transition variables (one of which may be time), while

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[1] The original "Taylor Rule" assumes that the US federal funds rate is raised by 1.5 percentage points for each 1 percentage point increase in inflation. An increase in the interest rate of that magnitude would raise real interest rates and help cool off the economy, hence reducing inflationary pressures. The rule also assumes that interest rates are reduced by 0.5 percentage point for each percentage point decline in real GDP below its potential. Such a reduction in the interest rate helps to mitigate a (growth cycle) recession and maintain price stability.

[2] It is not possible to employ central bank forecasts of inflation and output for any of the three countries over the entire period studied here. Real time forecasts prepared by the FED staff to inform US interest rate decisions are published, but these are available only with a lag of five years. Forecasts by the Bank of England for the UK are available only from 1992 and at a quarterly frequency, while we are not aware of any such published forecasts for Germany.

$$F(s_{it}) = [1 + \exp\{-\gamma_i (s_{it} - c_i) / \hat{\sigma}(s_{it})\}]^{-1} \qquad \gamma_i > 0,\ \ i = 1, 2. \qquad (3)$$

The disturbances $u_t$ are assumed to be *i.i.d.*(0, $\sigma^2$). For a previous application of this type of two-transition model, see Sensier, Osborn and Öcal (2002).

A feature of the present application is that we allow time to be one of the transition variables in (2), so that our models are able to capture evolution (or structural change) in the coefficients of the interest rate reaction function. This approach to modelling nonlinearity and structural change through STR models is discussed in some detail by Lundberg *et al*. (2003). Indeed, the models they consider have the form

$$r_t = \alpha_1' w_t\, (1 - F_1(t))(\,1 - F_2(s_t)) + \alpha_2' w_t\, F_1(t)(\,1 - F_2(s_t)) +$$
$$\alpha_3' w_t\, (1 - F_1(t))F_2(s_t) + \alpha_4' w_t\, F_1(t)\, F_2(s_t) + u_t \qquad (4)$$

so that one transition function is associated with time. However, we consider time and a range of explanatory variables as potential transition variables. Through this approach, we are able to examine and compare the evidence in favour of nonlinearity and time evolution.

Note also that, for given transition variables $s_{1t}$, $s_{2t}$, (2) is a restricted version of (4), where the $p$ restrictions imposed imply that $\alpha_1 - \alpha_2 - \alpha_3 + \alpha_4 = 0$. Given the relatively small numbers of observations we have available in some "regimes" (see the discussion below), we prefer to use the more parsimonious model in (2). Nevertheless, the separate examination of regimes implied in (4) provides a useful tool for the exposition of the models below.

The logistic function of (3) is attractive in our context, since it is a monotonically increasing function of $s_{it}$, and hence (depending on the transition variable) can capture, for example, effects of the business cycle or changes in interest rate responses by the central bank over time. Through the parameter $\gamma_i$, the transition between the two regimes $F(s_{it}) = 0$ and $F(s_{it}) = 1$ can be smooth (for relatively small $\gamma_i$) or abrupt, of the threshold form (large $\gamma_i$). Finally, the location of the transition between these regimes is given by the threshold parameter $c_i$, with the property that it captures the central point of the transition where $F(c_i) = 0.5$. As recommended by Teräsvirta (1994), the exponent of $F$ in (3) is standardised using the sample standard error of the transition variable.

While (2) represents the outcome of our modelling procedure, we do not commence with the proposition that two transition functions will be required to model the interest rate reaction functions for each country. Rather, we start from a linear specification and (in addition to conventional diagnostic tests) test for nonlinearity and time-variation in the coefficients. When required, we then consider a single transition model and, based on tests applied to this model, move to a two-transition model if this is justified. For all three countries this procedure led to two-transition models of the form of (2). The procedure used is outlined in the next subsection, with further details in the Appendix.

### 2.2 Selection of Explanatory and Transition Variables

Our modelling commences from a general version of the linear specification of (1). This initial general linear model contains three lags of each explanatory variable (except the constant), with three lags of interest rate also included. Individual lagged variables are then eliminated one by one (according to the lowest *t*-ratio) in order to obtain the linear model that minimises the Akaike Information Criterion (AIC). This specific linear model then provides the vector of explanatory variables $w_t$ in (1). However, this procedure was modified when the inflation gap was eliminated from $w_t$. In that case, due to the central role of inflation in monetary policy, we retained at least one lag of the inflation gap to ensure that it was considered in the nonlinear modelling (see the discussion of the US in Section 3.1 below).

Having selected the specific linear model, we then examine evidence of nonlinearity by considering each of the variables in $w_t$ as a possible (single) transition variable. In addition, we also add quarterly, bi-annual and annual differences of the interest rates to the set of possible transition variables, with these examined only at a lag of one month. These latter variables are considered in order to capture possible nonlinear effects associated with tighter versus looser monetary policy, where it is plausible that the central bank acts differently when interest rates are increasing or decreasing. Possible structural change is examined by considering time as a potential transition variable.

Our procedure considers each of these potential transition variables through both a test for significant nonlinearity and a grid search that estimates a range of nonlinear and time-varying models. When statistically significant evidence of nonlinearity and/or temporal instability is found, we estimate a single-transition STR model using the variable yielding lowest residual sum of squares in this grid search as the transition variable $s_{1t}$. This nonlinear model is refined by the elimination of individual variables (from $w_t$ and $Fw_t$) in order to minimise AIC.

When a two-transition model is specified, the first transition variable ($s_{1t}$) is taken as given, and a grid-search is undertaken over all other potential transition variables to identify the second ($s_{2t}$). To re-check the selection of $s_{1t}$, the transition variable $s_{2t}$ is then taken as given and a corresponding search is made to select $s_{1t}$. When these two searches do not deliver the same results, the two transition model is based on the transition variable pair ($s_{1t}$, $s_{2t}$) that delivers the lower residual sum of squares in the grid search.

### 2.3 Sample Periods and Data

Our modelling uses monthly data. The short-term interest rate is the Federal Funds Rate for the US, the money market rate for Germany and the Treasury bill yield for the UK[3]. All models are estimated using data from January 1984. The sample periods for the US and UK end in December 2002. For Germany, however, our series ends in December 1998, as interest rates have been set by the European Central Bank in relation to the Euro Area from the beginning of 1999.

Figure 1 presents graphs of the interest rates series used in modelling. One feature of those graphs is the distinctive pattern of German interest rates, which peak in 1993 compared to peaks of 1989 or 1990 for the US and UK. This distinctive temporal pattern for Germany is discussed further in the next section.

Because of data availability at the monthly frequency, the seasonally adjusted industrial production index is used to construct the output gap (OGAP) for the US and Germany. For the UK, we have available a monthly series for real gross domestic product[4], and this is employed in our analysis. In all cases, we apply the Hodrick-Prescott

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[3] In the IFS country pages, the interbank rate for UK is referred as the money market rate with the same line number. However, this variable is measured as the last Friday of the month, in contrast to the monthly averages for Federal Funds Rate and the German money market rate, resulting in more erratic variation. As Nelson (2000) discusses, the interest rate used as the Bank of England instrument has varied over time, and we follow Nelson in modelling the Treasury bill rate.
[4] This series is constructed by the National Institute of Economic and Social Research, and we are grateful to them for making it available to us, see Salazar, *et al* (1997) for further details.

(HP) filter in order to obtain the output gap series[5]. For the UK the annual percentage change of the retail price index measures inflation. For the US and Germany, our inflation measure is the annual change in the logarithm of the consumer price index multiplied by 100. The inflation target is measured by the published target values in the Bundesbank annual reports for Germany. For the US, it is calculated as the sample average of the actual inflation since data on target inflation are not available. For the UK, an inflation target of 2.5 percent has applied since 1997. Prior to this date, we compute the target as the centred two-year moving average of actual inflation[6]. The inflation gap (INFGAP) is then calculated as the difference between inflation and the target inflation series.

The real effective exchange rate index is defined as a nominal effective exchange rate index adjusted for relative movements in national prices, and this variable is used in first difference form. With the exception of UK monthly GDP, these data are taken from the International Financial Statistics (IFS) database of the International Monetary Fund, using the relevant country tables. World commodity price inflation is computed from the world commodity price index (from the IFS world table), converted to a percentage inflation measure as 100 times the first difference of the logarithm; this is denoted by $\Delta$WCP.

## 3. Results

The results are discussed below first for the linear models, and then for the preferred two-transition specifications. Results are not discussed in detail for the intermediate single transition models, although these can be found in the Appendix.

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[5]The output gap is defined as the difference between the level of output and the targeted level of output, which is assumed to be given by potential output. As in much of the literature, we use the HP filter to measure the long-run equilibrium (potential) level of output, with the output gap measured as the difference between actual output and this value. The HP filter (with a parameter of 126400, as suggested by Ravn and Uhlig, 2002) is applied to the monthly series.

[6] We experimented with various potential inflation target series for the period before 1997. However, some possibilities, such as sub-periods based on monetary policy regimes (see Nelson, 2000) result in discrete changes in the target, and hence in the inflation gap series, which we consider implausible.

### 3.1 Linear Models

As well as being of interest in their own right, the linear models play an important role in our nonlinear modelling procedure, since only the specific lags of the explanatory variables retained in the linear model are considered in the subsequent nonlinear specifications.

The estimated linear models that result from the procedure outlined above are shown in Table 1. In all three cases, a cursory examination of the results shows dynamics that effectively imply the presence of a unit root, with the sum of the autoregressive coefficients being close to unity. However, since such behaviour could be a consequence of unmodelled structural breaks or nonlinearity, we put this aside for the moment.

The linear models for Germany and the UK, shown in the final two columns of Table 1, are very similar. In both cases, the inflation gap at a one month lag has a significant (at 5 percent) and positive impact on interest rates, with a negative and significant effect after a further one or two lags, suggesting that the model might be reparameterised as one in the change in the inflation gap. However, as we prefer not to impose such restrictions at this early stage of the modelling procedure, we retain the specifications shown in the table for both countries. Also, the output gap appears only at lag one in each case, with a positive and significant coefficient, implying that the output gap plays an important role in setting interest rates. Finally, it should be noted that although world commodity price inflation and the real exchange rate were considered in the initial general model, neither appears in the specific linear model for either country.

Results for the linear US model are also presented in Table 1, and these are somewhat different from the other two countries. In this case, selection of variables based on minimum AIC led to a model without the inflation gap. As noted in section 2, we wish to retain a possible role for the inflation gap in our nonlinear modelling, due to its central role in monetary policy. Lags 1 and 3 of this variable are included in the US model of Table 1, since lag 3 was the most significant individual lag, and when this was included the lag 1 coefficient had the *a priori* anticipated positive sign.

In addition to interest rate dynamics (captured by two lags of the interest rate) and the inflation gap, two lags of the output gap and one lag of world commodity price inflation are included for the US. The coefficients of the output gap suggest that it may be the change in this variable that plays a role, rather than the level, but (once again) we do not wish to restrict the coefficients at this early stage of the analysis. The three month lag on world commodity price inflation implies that there is a delay before US monetary policy reacts to such inflation.

Diagnostics for these linear models are included in Table 1 in the form of *p*-values. Although the autocorrelation test is significant for Germany, autocorrelation in US and UK interest rates is satisfactorily accounted for by these models. The evidence of severe non-normality is, perhaps, not surprising for interest rates. ARCH effects are also apparent in the residuals of the linear models, but this may be due to unmodelled nonlinearity or structural change. The parameter constancy and nonlinearity diagnostics examine the possibility that time (for parameter constancy) and each explanatory variable of the model is the potential transition variable in a single transition STR model. It is clear that parameter constancy is strongly rejected in all cases, while evidence of nonlinearity at the 1 percent significance level or lower is also uncovered for all three countries.

It is unclear from these tests whether it is appropriate to allow time variation in the coefficients or nonlinearity, or both. For the US, in particular, not only is constancy rejected at the 0.1 percent significance level, but also nonlinearity is indicated (at this significance level) in relation to the lags of both interest rates and the output gap, together with commodity price inflation.  For Germany and the UK, the nonlinearity tests point particularly to the inflation gap as the potential transition variable, while parameter constancy is also rejected at a significance level of 1 percent. To resolve this question, we rely primarily on our grid search procedure to select the transition variable(s), with the resulting nonlinear models discussed below.

### 3.2 Nonlinear Models

Single transition models were estimated for all three countries, but the diagnostics of these models were not satisfactory (see Appendix Table A.2). In particular, the US and UK models continue to evidence parameter non-constancy, with *p*-values around the 1 percent significance level[7]. The single transition model for Germany is more satisfactory

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[7] We also present the single transition models with time as the transition variable for the US and UK for comparison.

in this respect, but still fails to account for the nonlinearity associated with the inflation gap. Therefore, here we discuss only our preferred two-transition function models.

The estimated two-transition models are shown in Table 2, with the corresponding transition functions in Figures 2, 3 and 4 for the US, UK and Germany respectively[8]. Corresponding to the evidence of parameter non-constancy in the linear models, our model specification procedure (see the Appendix) selects time as a transition variable for both the US and the UK. Although this is not the case for Germany, Figure 4 indicates that one of the selected transition variables (namely $\Delta_{12}r_{t-1}$) implies a nonzero transition function primarily for a relatively short period around 1989-1990. We believe that this transition may be detecting monetary policy in Germany specific to the period of reunification, and in this sense may also reflect a form of parameter non-constancy, though of a temporary form. The ordering of the two transition functions in Table 2 is arbitrary, but we denote those associated with these time effects as the first transition in each case.

Figure 2a shows that the time transition for the US implies that the interest rate reaction function coefficients change rather abruptly in 1985, soon after the beginning of our sample period. For the UK, on the other hand, the model implies that the parameters evolve smoothly during the second half of the 1980s (see Figure 3a). It is noteworthy that this evolution is effectively complete prior to the explicit adoption of inflation targeting for the UK in 1992[9]. In each case, the number of observations associated with one of these regimes is relatively small[10].

For both of these countries, Table 2 shows that the estimated intercept shifts down by around 2½ percentage points when $F_1(t) = 1$, which indicates (for given inflation and output gaps) lower interest rates from the mid-or late-1980s. In the case of the US, interest rate dynamics captured in the model also change with the time transition, while

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[8] At the final stage, we have applied some restrictions to the models specified by our procedure. For the US we restrict the two coefficients on world commodity prices to be equal in magnitude and opposite in sign, and this is accepted with a p-value of 0.27. For Germany we remove the inflation gap in the linear part of the model as this is insignificant, with a p-value of 0.20. In both of these cases, the restrictions improve AIC.

[9] It is interesting that, in an investigation of the information content of the term structure of interest rates for forecasting future inflation in the UK, Bårdsen, Becker and Hurn (2004) find that the structural break occurs during 1990, rather than with the beginning of inflation targeting in 1992.

[10] For this reason, estimation of the model in the form of (4), thereby directly estimating the coefficients of the four regimes implied by the values of $F_1$ and $F_2$, is impractical.

the role of world commodity prices disappears after 1985. It is also notable that the output gap coefficients change for both countries, with the apparently perverse negative coefficient for the UK in the upper part of Table 2 being only a temporary phenomenon associated with the early part of the period.

In the case of Germany, the first transition function changes only the intercept and interest rate dynamics. In particular, the significantly higher intercept for 1988-1990, compared with months when $F_1(\Delta_{12} r_{t-1}) = 0$, indicates that the output gap and the inflation gap do not explain the relatively high interest rates in Germany over this reunification period.

To focus on the implications of these models for the recent period, Table 3 shows the implied coefficients of the models when $F_1(t) = 1$ for the US and UK, while $F_1(\Delta_{12} r_{t-1}) = 0$ for Germany. Thus, we consider the period when the time transition has been completed for the US and UK, while the temporary effects captured by the first transition for Germany do not apply. Given these specific values for the first transition function for each country, the table then illustrates the implications of $F_2 = 0$ versus $F_2 = 1$. Thus, Table 3 shows the estimated coefficients of Table 2 in the form of the coefficients of equation (4), by explicitly considering regimes implied by the estimated two-transition models. The relevant transition variable for this second function is, in each case, a one- or three-month difference of interest rates, with the transition function being (effectively) zero for interest rate declines; see the lower half of each of Figures 2 to 4. Therefore, we refer to $F_2 = 0$ as being declining interest rates, and $F_2 = 1$ as increasing interest rates.

Interest rate dynamics, as captured by the models for each of the three countries, are similar for the recent period when interest rates have been declining. Further, the inflation gap plays little or no role, with the coefficients for the US being of an unexpected negative sign and that for the UK significant at only the 10 percent level (see Table 2). On the other hand, the output gap has the expected positive sign at a one month lag in each case. Therefore, the models imply that during periods of declining interest rates, the output gap plays a role but (presumably because interest rate declines occur only when inflationary conditions are benign), the inflation gap is relatively unimportant.

At least for the UK and Germany, the past inflation gap becomes important for interest rate behaviour during periods of increasing interest rates. The signs and magnitudes of the UK coefficients suggest that the change in the inflation gap over the previous month is important in this case (see the lower part of Table 3), while for Germany both the level of the previous month and the change over two months play a role (since the coefficients of $INFGAP_{t-1}$ and $INFGAP_{t-3}$ can be reparameterised in terms of $INFGAP_{t-1}$ and $\Delta_2 INFGAP_{t-1}$). Further, the output gap plays a greater role here for the UK compared with periods of declining interest rates. It is also noteworthy that US and UK interest rate dynamics change substantially in periods of increasing versus declining interest rates.

Unfortunately, however, our US model is not plausible for periods of increasing interest rates, with both the inflation and output gaps having negative coefficients at a lag of one month. The reason for this may lie in the relatively small number of observations when this transition function is above (say) 0.5, so that relatively little information is available about behaviour in this regime (see the lower panel of Figure 2). In this context, the inevitable collinearity between the values of the transition function itself and other variables multiplied by this transition function is likely to lead to imprecise coefficient estimates.

One feature common to the models of Table 2 is that the dynamics of the lagged dependent variable imply behaviour that is close to nonstationarity; this is particularly clear when the models are written as in Table 3. Therefore, we do not attribute this near-nonstationary behaviour to nonlinearity or structural breaks. Nevertheless, it is also notable that our models do not account for all features of the interest rate series, with some evidence (at around the 5 percent significance level) of parameter instability remaining in the nonlinear specifications. On the other hand, only one nonlinearity test statistic in Table 2 is significant at 5 percent, so that the nonlinearity evident in Table 1 has been effectively accounted for within our models. Further, although not the case for the UK, the strong ARCH effects found in the linear models of Table 1 also disappear when temporal instability and nonlinearity in the reaction function is modelled for the US and Germany.

## 4. Concluding Remarks

Our examination of the evidence for nonlinearity and parameter instability in the interest rate reaction functions of the US, the UK and Germany has revealed substantial evidence that such features are important for interest rates over our sample period from 1984. Indeed, common across all our estimated models, nonlinearity is primarily associated with time and the dynamics of interest rates, rather than with past values of the output gap, the inflation gap or world commodity price inflation.

In the developing literature of nonlinear monetary policy rules, studies have almost exclusively focused on either the output gap or inflation in relation to target as the essential nonlinear feature. Interest rate dynamics have not been considered to be relevant and have typically simply been assumed constant over time. Similarly, most researchers assume that (nonlinear) interest rate policy has been constant in the period of relatively low interest rates since 1984. Our models indicate that such assumptions could lead to substantial misspecification.

Our models also point to further avenues of research in this area. In particular, despite allowing for nonlinearity and parameter non-constancy, there are indications that some unmodelled instability may remain in our models. To capture these effects, even greater attention may need to be paid to modelling changes in monetary policy over the period from the mid-1980s. However, in this context, it is difficult to distinguish effects due to coefficients which change as a function of time (associated with, for example, changing monetary policy) and those which change due to inherent nonlinearities in the interest rate reaction functions. We hope that further research will help to resolve this issue.

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**Table 1**
**Linear Interest Rate Models**

| Variable | US | UK | Germany |
|---|---|---|---|
| Constant | -0.017 | 0.100 | 0.053 |
| | (-0.285) | (1.00) | (0.709) |
| $r_{t-1}$ | 1.334 | 1.115 | 1.152 |
| | (21.2) | (16.8) | (15.6) |
| $r_{t-2}$ | -0.336 | -0.231 | -0.164 |
| | (-5.29) | (-2.35) | (-2.21) |
| $r_{t-3}$ | | 0.100 | |
| | | (1.51) | |
| $INFGAP_{t-1}$ | 0.030 | 0.306 | 0.080 |
| | (0.848) | (2.07) | (2.35) |
| $INFGAP_{t-2}$ | | -0.340 | |
| | | (-2.28) | |
| $INFGAP_{t-3}$ | -0.050 | | -0.072 |
| | (-1.44) | | (-2.15) |
| $OGAP_{t-1}$ | 0.121 | 0.082 | 0.022 |
| | (3.51) | (2.92) | (2.61) |
| $OGAP_{t-2}$ | -0.106 | | |
| | (-3.10) | | |
| $\Delta WCP_{t-3}$ | 2.279 | | |
| | (2.70) | | |
| **Summary Statistics** | | | |
| AIC | -2.966 | -1.405 | -3.101 |
| $R^2$ | 0.990 | 0.977 | 0.990 |
| $s$ | 0.223 | 0.488 | 0.209 |
| **Diagnostic Tests (*p*-values)** | | | |
| Autocorrelation | 0.336 | 0.806 | 0.008 |
| ARCH | 0.001 | 0.000 | 0.002 |
| Normality | 0.000 | 0.000 | 0.000 |
| Parameter constancy | 0.009 | 0.006 | 0.002 |
| Nonlinearity test for transition variable: | | | |
| $r_{t-1}$ | 0.005 | 0.213 | 0.054 |
| $r_{t-2}$ | 0.005 | 0.407 | 0.199 |
| $r_{t-3}$ | N/A | 0.443 | N/A |
| $INFGAP_{t-1}$ | 0.036 | 0.437 | 0.001 |
| $INFGAP_{t-2}$ | N/A | 0.000 | N/A |
| $INFGAP_{t-3}$ | 0.278 | N/A | 0.010 |
| $OGAP_{t-1}$ | 0.000 | 0.030 | 0.634 |
| $OGAP_{t-2}$ | 0.000 | N/A | N/A |
| $\Delta WCP_{t-3}$ | 0.001 | N/A | N/A |

Notes: Values in parentheses are *t*-values. Lagrange multiplier tests for autocorrelation and heteroscedasticity consider processes of order 6 under the alternative hypotheses. The parameter constancy/nonlinearity test is that of Luukkonen, Saikkonen and Teräsvirta (1988), applied using time or an explanatory variable of the model. N/A is not applicable, as the corresponding variable does not appear in the model.

**Table 2**
**Nonlinear Interest Rate Models**

| Variable | US | UK | Germany |
|---|---|---|---|
| Constant | 2.437 (2.95) | 2.619 (5.43) | 0.028 (0.57) |
| $r_{t-1}$ | 1.098 (10.52) | 1.310 (14.49) | 1.310 (14.04) |
| $r_{t-2}$ | -0.375 (-5.39) | -0.555 (-5.87) | -0.321 (-3.51) |
| $INFGAP_{t-1}$ | | 0.157 (1.84) | |
| $INFGAP_{t-3}$ | -0.085 (-2.36) | | |
| $OGAP_{t-1}$ | 0.163 (4.89) | -0.424 (-3.51) | 0.018 (2.17) |
| $\Delta WCP_{t-3}$ | 17.63 (5.41) | | |
| $F_1$ | -2.549 (3.12) | -2.425 (-5.08) | 0.850 (2.29) |
| $F_1 \times r_{t-1}$ | 0.289 (3.37) | | -0.902 (-4.98) |
| $F_1 \times r_{t-2}$ | | | 0.831 (4.81) |
| $F_1 \times r_{t-3}$ | | 0.216 (4.81) | |
| $F_1 \times INFGAP_{t-1}$ | 0.055 (1.58) | | |
| $F_1 \times OGAP_{t-1}$ | | 0.538 (4.42) | |
| $F_1 \times OGAP_{t-2}$ | -0.181 (-5.05) | | |
| $F_1 \times \Delta WCP_{t-3}$ | -17.63 (-5.41) | | |
| $s_{1t}$ | *Time* | *Time* | $\Delta_{12}r_{t-1}$ |
| $\gamma_1$ | 1082 (0.02) | 13.16 (1.84) | 3.950 (1.56) |
| $c_1$ | 14.14 (1.83) | 33.65 (5.04) | 1.231 (3.69) |
| $F_2$ | 1.034 (1.62) | | 1.011 (2.19) |
| $F_2 \times r_{t-1}$ | -0.731 (-1.48) | -0.687 (-4.35) | |
| $F_2 \times r_{t-2}$ | 0.605 (1.30) | 0.706 (4.17) | -0.212 (-2.40) |
| $F_2 \times INFGAP_{t-1}$ | -0.437 (-1.49) | 2.879 (6.66) | 0.654 (3.08) |
| $F_2 \times INFGAP_{t-2/3}$ | 0.595 (1.67) | -3.296 (-8.70) | -0.352 (-2.26) |
| $F_2 \times OGAP_{t-1}$ | -0.688 (-1.72) | 0.231 (3.28) | |
| $F_2 \times OGAP_{t-2}$ | 0.923 (1.95) | | |
| $s_{2t}$ | $\Delta_3 r_{t-1}$ | $\Delta r_{t-1}$ | $\Delta_3 r_{t-1}$ |
| $\gamma_2$ | 2.255 (2.65) | 927.9 (0.005) | 3.134 (2.66) |
| $c_2$ | 0.757 (2.62) | 0.321 (2.77) | 0.460 (3.94) |
| AIC | -3.289 | -1.705 | -3.337 |
| $R^2$ | 0.993 | 0.985 | 0.993 |
| $s$ | 0.185 | 0.411 | 0.181 |
| **Diagnostic Tests (*p*-values)** | | | |
| Autocorrelation | 0.615 | 0.192 | 0.555 |
| ARCH | 0.492 | 0.000 | 0.952 |
| Normality | 0.000 | 0.000 | 0.000 |
| Parameter Constancy | 0.042 | 0.047 | 0.040 |
| Additional nonlinearity test for transition variable: | | | |
| $r_{t-1}$ | 0.492 | 0.248 | 0.187 |
| $r_{t-2}$ | 0.334 | 0.132 | 0.192 |
| $r_{t-3}$ | N/A | 0.680 | N/A |
| $INFGAP_{t-1}$ | 0.022 | 0.664 | 0.064 |
| $INFGAP_{t-3}$ | 0.090 | 0.167 | 0.147 |
| $OGAP_{t-1}$ | 0.382 | 0.197 | 0.774 |
| $OGAP_{t-2}$ | 0.652 | N/A | N/A |
| $\Delta WCP_{t-3}$ | 0.746 | N/A | N/A |

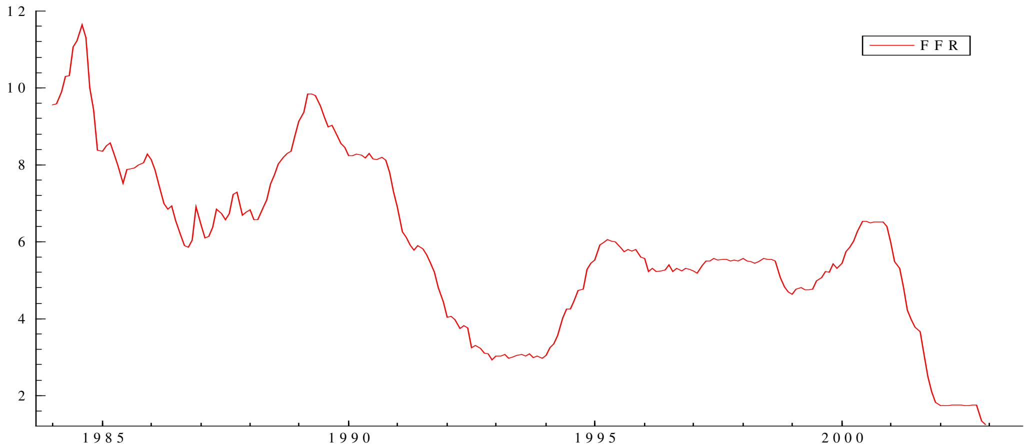
Notes: Values in parentheses are *t*-values. Lagrange multiplier tests for autocorrelation and heteroscedasticity consider processes of order 6 under the alternative hypotheses. Diagnostic tests for autocorrelation, parameter constancy and additional nonlinearity are those proposed by Eitrheim and Teräsvirta (1996).

**Table 3**
**Interest Rate Responses for Recent Period**

| Variable | US | UK | Germany |
|---|---|---|---|
| Declining interest rates ($F_2 = 0$) | | | |
| Constant | -0.112 | 0.194 | 0.878 |
| $r_{t-1}$ | 1.387 | 1.310 | 0.408 |
| $r_{t-2}$ | -0.375 | -0.555 | 0.510 |
| $r_{t-3}$ | | 0.216 | |
| $INFGAP_{t-1}$ | 0.055 | 0.157 | |
| $INFGAP_{t-3}$ | -0.085 | | |
| $OGAP_{t-1}$ | 0.163 | 0.114 | 0.018 |
| $OGAP_{t-2}$ | -0.181 | | |
| Increasing interest rates ($F_2 = 1$) | | | |
| Constant | 0.922 | 0.194 | 1.889 |
| $r_{t-1}$ | 0.656 | 0.623 | 0.408 |
| $r_{t-2}$ | 0.230 | 0.151 | 0.298 |
| $r_{t-3}$ | | 0.216 | |
| $INFGAP_{t-1}$ | -0.382 | 3.036 | 0.654 |
| $INFGAP_{t-2}$ | | -3.296 | |
| $INFGAP_{t-3}$ | 0.510 | | -0.352 |
| $OGAP_{t-1}$ | -0.525 | 0.345 | 0.018 |
| $OGAP_{t-2}$ | 0.742 | | |

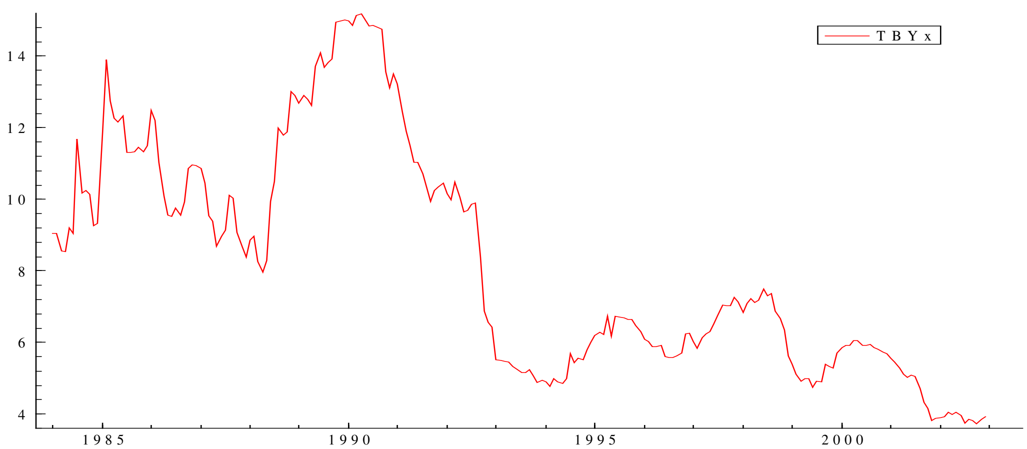
Notes The coefficients are derived from the estimated models of Table 2, with $F_1(t) = 1$ for the US and UK, and $F_1(\Delta_{12} r_{t-1}) = 0$ for Germany. The representation shows the implied coefficients in the separate regimes, as in equation (4).

**Figure 1. Graphs of Interest Rate Variables**




US Federal Funds Rate

UK Treasury Bill Yield




German Money Market Rate

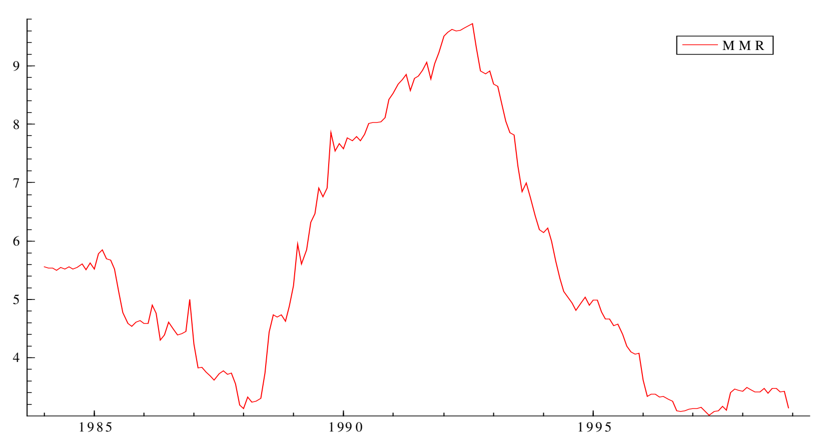


**Figure 2a. Time Transition Function for the US**

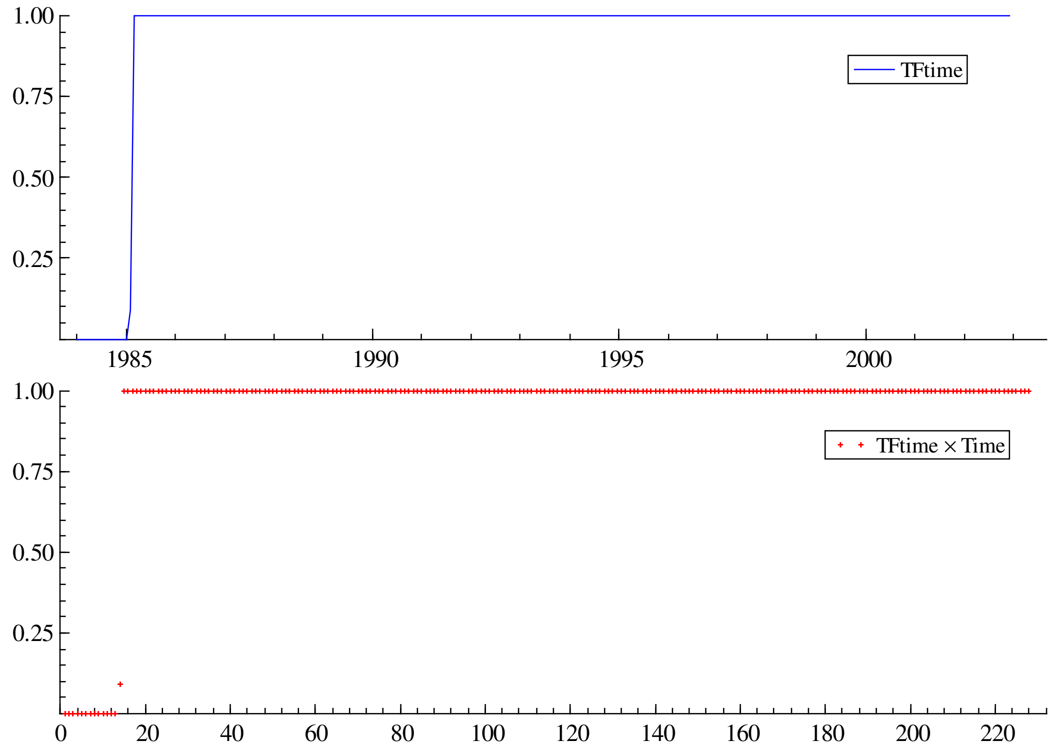



**Figure 2b. Interest Rate Transition Function for the US**

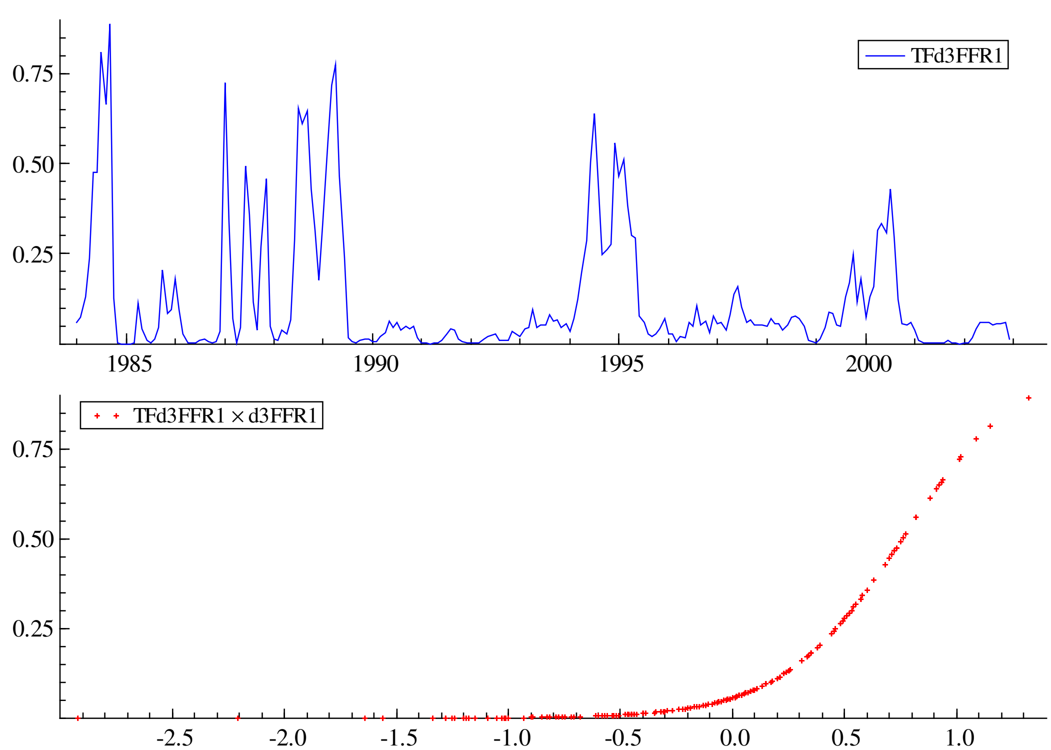


**Figure 3a. Time Transition Function for the UK**

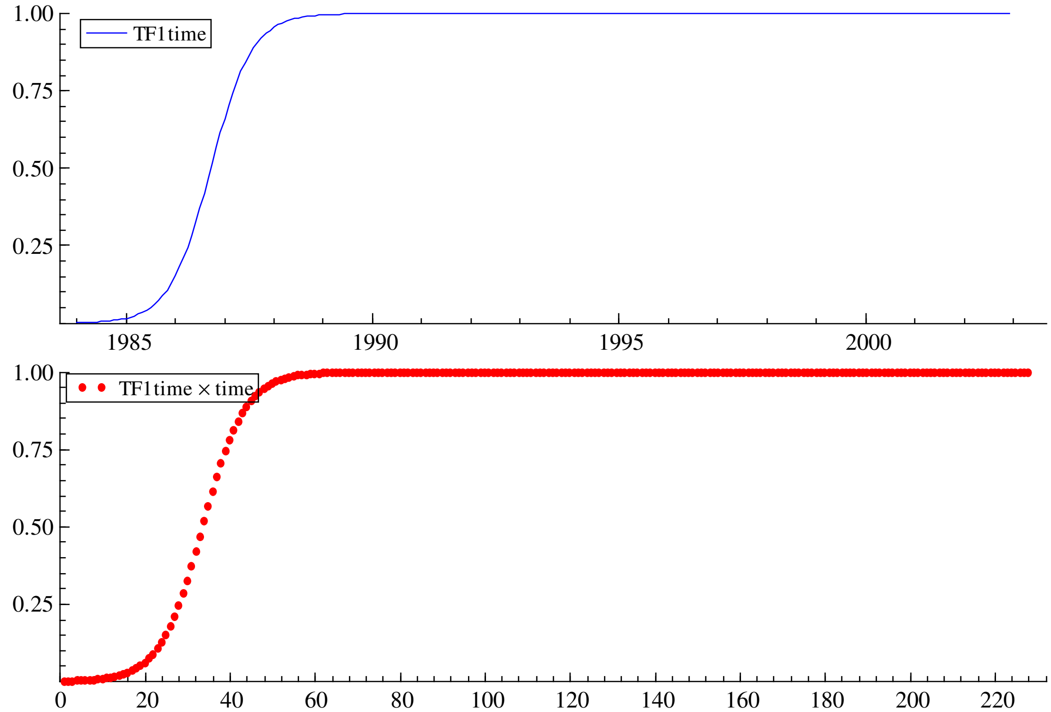



**Figure 3b. Interest Rate Transition Function for the UK**

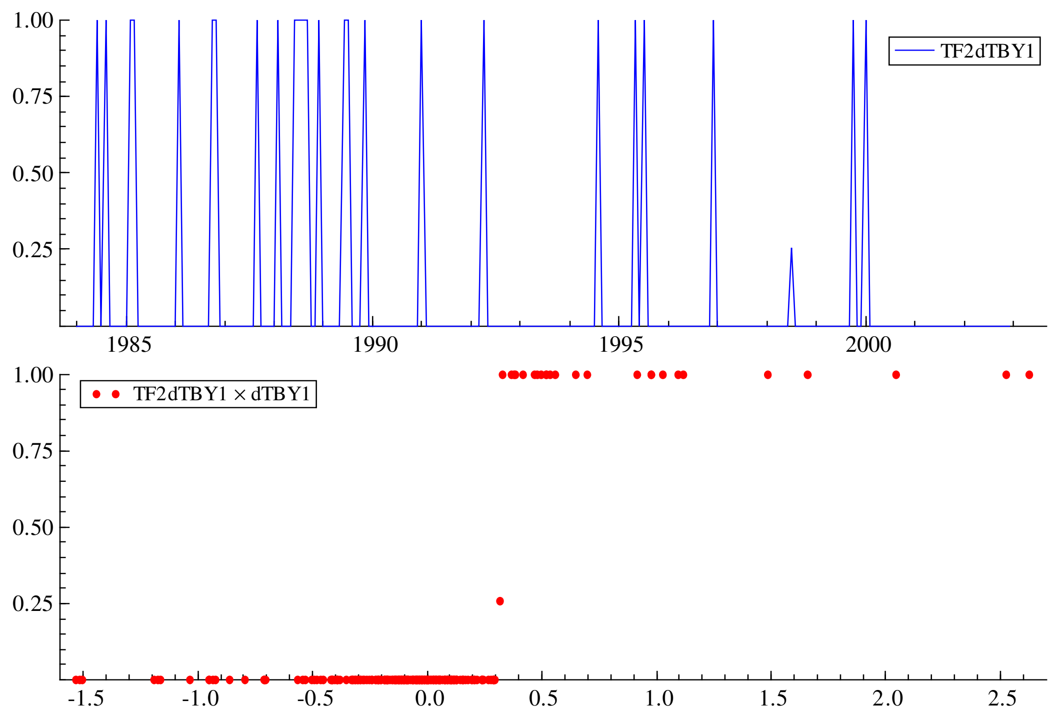


**Figure 4a. First Interest Rate Transition Function for Germany**

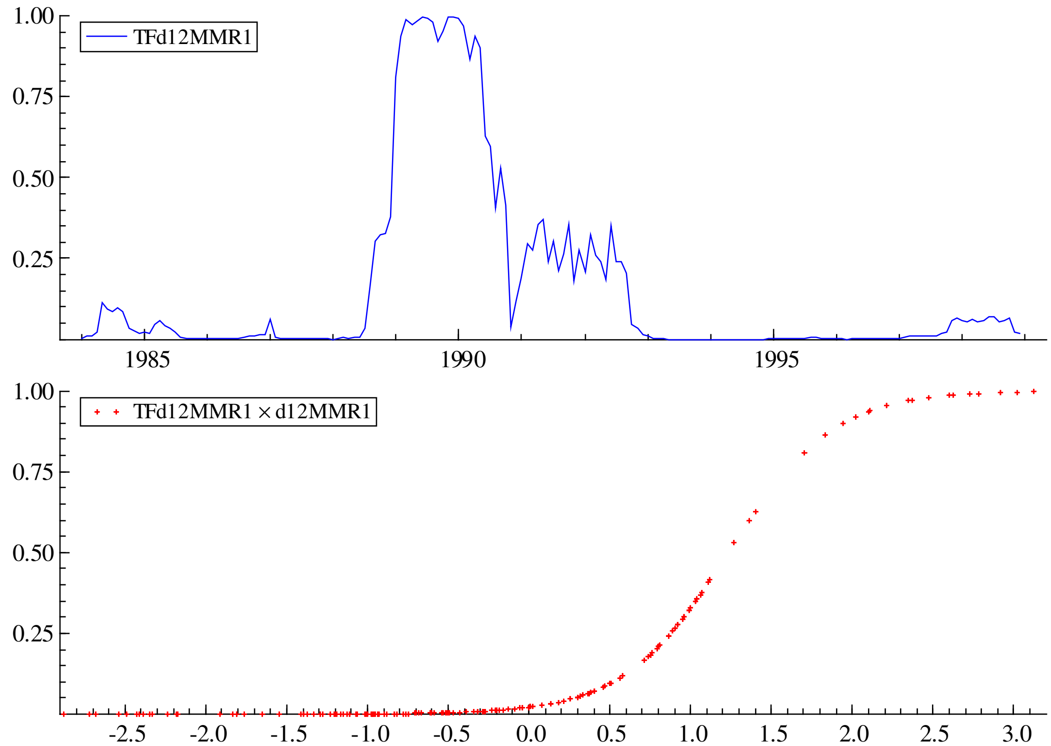
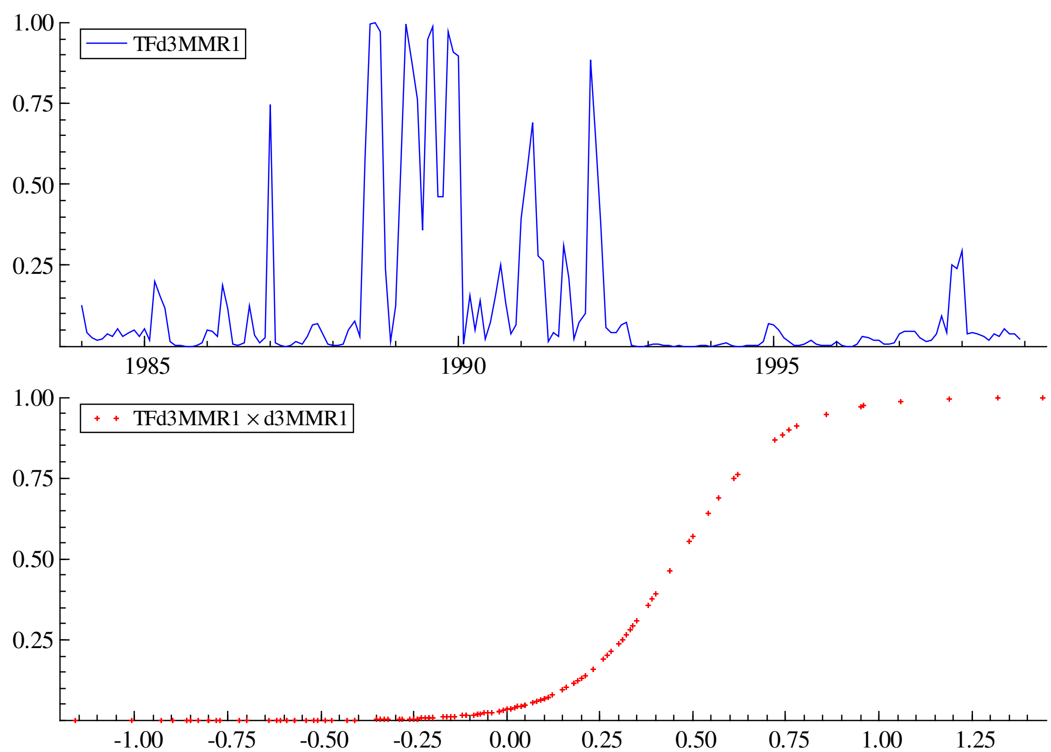



**Figure 4b. Second Interest Rate Transition Function for Germany**



## Appendix

## Modelling Methodology and Additional Results

Here we outline important aspects of the estimation and evaluation of the STR models. In particular, details of specification, estimation and diagnostic checking are addressed. Our procedure largely follows Teräsvirta (1994, 1998). However, we rely more extensively on grid search methods in order to select the transition variable(s) and on ordinary least squares (OLS) for initial estimation of the STR coefficients. The procedure followed here is effectively the same as in Sensier *et al.* (2002).

In the case of a single transition, the STR model is defined (Teräsvirta 1994, 1998) as:

$$r_t = \beta_0' w_t + \beta_1' w_t F(s_t) + u_t \qquad \text{(A.1)}$$

where, as in the text, the logistic function is used to define $F(s_t)$. However, prior to estimating such a model, we test linearity against the STR specification. It is difficult to test linearity versus nonlinearity directly in (A.1), due to the lack of identification of the parameters under the linearity null hypothesis. However, a third order Taylor series approximation to $F(s_t)$ yields a test of linearity against STR nonlinearity as a test of the null hypothesis $\delta_{2j} = \delta_{3j} = \delta_{4j} = 0$ ($j = 1,\ldots\ldots, m$) in the artificial regression

$$y_t = \delta_0 + \delta_1' w_t + \delta_2' w_t s_t + \delta_3' w_t s_t^2 + \delta_4' w_t s_t^3 + v_t \qquad \text{(A.2)}$$

(Luukkonen, Saikkonen and Teräsvirta, 1988). In practice this is conducted as an F-test for variable deletion. Each explanatory variable in $w_t$ (excluding the intercept) is considered as the possible transition variable $s_t$. To test parameter constancy, time is also considered as a transition variable. The results are shown in the diagnostic tests for the linear model reported in Table 1.

Having established the presence of nonlinearity and/or parameter non-constancy, the transition variable ($s_t$) in (A.1) is selected using a grid search procedure and applying OLS regression. Each explanatory variable in $w_t$ and time, together with lagged interest rate changes (see section 2.2), are considered as the potential $s_t$. Our grid search uses 150 values of $\gamma$ and 40 values of $c$ within the observed range of each variable considered, to define a range of transition functions $F(s_t)$. For each $s_t$, $\gamma$ and $c$, values for $F(s_t)$ are computed and OLS is then applied to (A.1). The potential transition variable yielding the minimum residual sum of squares (RSS) is considered as the transition variable $s_t$.

Results of the grid search (shown in each case as the six potential transition variables yielding the lowest values of the RSS) are presented in Appendix Table A.1. In addition to the grid search results, we also present the $p$-value for a linearity test with this variable taken as $s_t$. It is obvious from the results that selection using the smallest $p$-value, as advocated by Teräsvirta (1994), does not necessarily lead to the same transition variable as the grid search. We favour the grid search approach as this is directly based on a best fit criterion for the nonlinear model. This selects $r_{t-1}$, $\Delta_6 r_{t-1}$ and $\Delta_3 r_{t-1}$ as the transition variables for the US, UK and Germany respectively.

Having selected the transition variable, we refine the STR model of (A.1) employing OLS, conditional on the transition function that yielded minimum RSS. We adopt a general-to-specific approach, with the linear model of Table 1 defining the elements of $w_t$. Individual variables (including terms deriving from $Fw_t$) are dropped sequentially using the smallest $t$-ratio, to obtain the model that minimises AIC. The STR model is then estimated by nonlinear least squares, including the transition function parameters $c$ and $\gamma$, using the previous "linear" STR estimates to provide initial values for the nonlinear estimation. The $\gamma$ and $c$ values from the nonlinear estimation are compared with those derived from the grid search to ensure they do not substantially differ.

The resulting estimated single transition models for each country are reported in Appendix Table A.2. Note that the US and UK models continue to show strong evidence of parameter non-constancy, while there is evidence of nonlinearity for Germany in relation to the inflation gap. Due to the evidence of parameter non-constancy, and for comparison with the two-transition models, Table A.2 also presents single-transition models for the US and the UK based on a time transition. In terms of goodness of fit criteria, the two models for each country are very similar, indicating that it is difficult to statistically distinguish between time change and nonlinearity in this context where the properties of interest rates have changed over time. However, the time transition models in Table A.2 are also unsatisfactory, showing evidence of both nonlinearity and additional time non-constancy. Therefore we develop two transition function models.

As outlined in Section 2, we initially take the transition variable selected from the single transition grid search as $s_{1t}$ and conduct a grid search for the second transition variable over all other variables in $w_t$. For a given potential $s_{2t}$, this grid search is conducted over values for $\gamma_1$, $\gamma_2$, $c_1$ and $c_2$ (that is, over the slope and location parameters for both transitions). To investigate whether a different combination of transition variables may yield a lower RSS, we then use the selected $s_{2t}$ variable and repeat the grid search procedure to select $s_{1t}$ (again searching over the slope and location parameters for both transitions). The pair of variables yielding the lowest RSS overall are employed in the two transition model. Results from the two transition grid search are shown in Appendix Table A.3, for the six combinations yielding the lowest RSS values. It might be noted that the variable selected as $s_t$ from the single transition grid search for each of the US and UK ($r_{t-1}$ and $\Delta_6 r_{t-1}$ respectively) does not lead to the lowest RSS in Appendix Table A.3, and hence does not appear as either $s_{1t}$ or $s_{2t}$ in the two-transition specification of Table 2.

**Appendix Table A.1**
**Grid Search for Single Transition Models**

| Transition Variable ($s_t$) | Grid Search Results $\gamma$ | c | RSS |
|---|---|---|---|
| **US** | | | |
| $r_{t\text{-}1}$ | 150 | 9.268 | 8.671 |
| $t$ | 59 | 16 | 8.831 |
| $\Delta_3 r_{t\text{-}1}$ | 110 | 0.685 | 8.902 |
| $OGAP_{t\text{-}1}$ | 150 | 2.226 | 9.008 |
| $OGAP_{t\text{-}2}$ | 150 | 2.118 | 9.039 |
| $\Delta_{12} r_{t\text{-}1}$ | 117 | 1.625 | 9.079 |
| **UK** | | | |
| $\Delta_6 r_{t\text{-}1}$ | 150 | 0.295 | 43.797 |
| $INFGAP_{t\text{-}2}$ | 33 | -0.610 | 44.069 |
| $\Delta r_{t\text{-}1}$ | 58 | 0.307 | 44.097 |
| $t$ | 13 | 35.7 | 44.176 |
| $\Delta_3 r_{t\text{-}1}$ | 150 | 1.047 | 45.978 |
| $r_{t\text{-}1}$ | 150 | 11.535 | 46.503 |
| **Germany** | | | |
| $\Delta_3 r_{t\text{-}1}$ | 150 | 0.416 | 5.631 |
| $\Delta_{12} r_{t\text{-}1}$ | 3 | 1.190 | 6.015 |
| $\Delta_6 r_{t\text{-}1}$ | 150 | 0.594 | 6.090 |
| $\Delta r_{t\text{-}1}$ | 4 | 0.246 | 6.218 |
| $t$ | 7 | 105.4 | 6.535 |
| $r_{t\text{-}1}$ | 150 | 6.753 | 6.695 |

Note: For each country, results are shown for the six potential transition variables considered that yield the lowest values for the residual sum of squares in the single transition grid search.

## Appendix Table A.2
## Estimated Single Transition Models

| Variable | US | US | UK | UK | Germany |
|---|---|---|---|---|---|
| Constant | -0.048 (-0.84) | -1.275 (-1.40) | 0.100 (1.08) | 3.709 (5.44) | 0.014 (0.334) |
| $r_{t-1}$ | 1.296 (23.12) | 1.391 (14.02) | 1.508 (16.77) | 0.819 (10.15) | 1.264 (15.16) |
| $r_{t-2}$ | -0.292 (-5.18) | -0.284 (-4.63) | -0.514 (-5.83) | -0.180 (-2.80) | -0.271 (-3.26) |
| $\text{INFGAP}_{t-1}$ | | | | 1.327 (4.01) | |
| $\text{INFGAP}_{t-2/3}$ | -0.029 (-1.90) | -0.036 (-2.31) | | -1.097 (-3.34) | |
| $\text{OGAP}_{t-1}$ | 0.138 (4.48) | 0.123 (3.84) | | -0.313 (-2.58) | 0.019 (2.66) |
| $\text{OGAP}_{t-2}$ | -0.108 (-3.51) | | | | |
| $\Delta\text{WCP}_{t-3}$ | | 25.60 (9.48) | | | |
| $F_1$ | -1.294 (-1.59) | 1.208 (1.33) | | -3.569 (-5.17) | 1.644 (6.901) |
| $F_1 \times r_{t-1}$ | 0.130 (1.60) | -0.099 (-1.11) | -0.686 (-5.60) | 0.338 (5.06) | -0.773 (-5.07) |
| $F_1 \times r_{t-2}$ | | | 0.658 (5.37) | | 0.525 (3.434) |
| $F_1 \times \text{INFGAP}_{t-1}$ | | -1.171 (-3.17) | | -1.171 (-3.17) | 0.583 (4.99) |
| $F_1 \times \text{INFGAP}_{t-2/3}$ | | 0.989 (2.71) | | 0.989 (2.71) | -0.313 (-2.90) |
| $F_1 \times \text{OGAP}_{t-1}$ | | | 0.245 (5.20) | 0.417 (3.37) | |
| $F_1 \times \text{OGAP}_{t-2}$ | | -0.096 (-3.04) | | | |
| $F_1 \times \Delta\text{WCP}_{t-3}$ | 22.20 (7.29) | -24.54 (-9.05) | | | |
| $s_t$ | $r_{t-1}$ | *Time* | $\Delta_6 r_{t-1}$ | *Time* | $\Delta_3 r_{t-1}$ |
| $\gamma_1$ | 433.9 (0.01) | 2170 (0.02) | 244.1 (0.52) | 11.62 (1.61) | 385.1 (0.01) |
| $c_1$ | 9.351 (13.76) | 12.49 (3.64) | 0.307 (13.92) | 37.1 (4.86) | 0.421 (1.06) |
| AIC | -3.156 | -3.168 | -1.556 | -1.515 | -3.320 |
| $R^2$ | 0.992 | 0.992 | 0.981 | 0.981 | 0.992 |
| $s$ | 0.202 | 0.200 | 0.452 | 0.456 | 0.185 |
| **Diagnostic Tests** (*p*-values) | | | | | |
| Autocorrelation | 0.083 | 0.175 | 0.723 | 0.273 | 0.180 |
| ARCH | 0.000 | 0.002 | 0.000 | 0.000 | 0.998 |
| Normality | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Parameter Constancy | 0.000 | 0.012 | 0.014 | 0.019 | 0.129 |
| Additional Nonlinearity Tests for Transition Variable: | | | | | |
| $r_{t-1}$ | 0.861 | 0.490 | 0.410 | 0.393 | 0.052 |
| $r_{t-2}$ | 0.967 | 0.658 | 0.429 | 0.475 | 0.096 |
| $\text{INFGAP}_{t-1}$ | 0.024 | 0.006 | 0.328 | 0.312 | 0.019 |
| $\text{INFGAP}_{t-2/3}$ | 0.038 | 0.034 | 0.132 | 0.000 | 0.064 |
| $\text{OGAP}_{t-1}$ | 0.646 | 0.541 | 0.045 | 0.026 | 0.468 |
| $\text{OGAP}_{t-2}$ | 0.438 | 0.427 | N/A | N/A | N/A |
| $\Delta\text{WCP}_{t-3}$ | 0.415 | 0.374 | N/A | N/A | N/A |

Notes: See Table 2. The lag 2/3 for the inflation gap (INFGAP) is two for the UK and 3 for the US and Germany. N/A is not applicable, as the corresponding variable does not appear in the model.

**Appendix Table A.3**
**Grid Search for Two Transition Models**

| **First Transition Function** | | | **Second Transition Function** | | | |
|---|---|---|---|---|---|---|
| $s_{1t}$ | $\gamma_1$ | $c_1$ | $s_{2t}$ | $\gamma_2$ | $c_2$ | **RSS** |
| **US** | | | | | | |
| $t$ | 30 | 16 | $\Delta_3 r_{t-1}$ | 4 | 0.336 | 7.232 |
| $t$ | 30 | 16 | $\Delta r_{t-1}$ | 1 | 0.144 | 7.264 |
| $r_{t-1}$ | 431 | 6.842 | $t$ | 30 | 16.00 | 7.548 |
| $t$ | 30 | 16 | $INFGAP_{t-1}$ | 15 | -1.675 | 7.936 |
| $r_{t-1}$ | 431 | 6.842 | $OGAP_{t-2}$ | 4 | 1.468 | 7.952 |
| $t$ | 30 | 16 | $OGAP_{t-2}$ | 30 | -1.132 | 7.964 |
| **UK** | | | | | | |
| $t$ | 12 | 35.70 | $\Delta r_{t-1}$ | 23 | 0.337 | 35.62 |
| $t$ | 12 | 16 | $INFGAP_{t-2}$ | 30 | -0.6099 | 35.72 |
| $t$ | 38 | 65.25 | $\Delta_6 r_{t-1}$ | 97 | 0.665 | 36.47 |
| $t$ | 31 | 35.70 | $\Delta_3 r_{t-1}$ | 1 | 0.9845 | 36.79 |
| $t$ | 27 | 16.00 | $\Delta_{12} r_{t-1}$ | 42 | 2.17 | 38.30 |
| $t$ | 40 | 104.6 | $r_{t-3}$ | 10 | 8.838 | 38.36 |
| **Germany** | | | | | | |
| $\Delta_3 MMR_{t-1}$ | 4 | 0.276 | $\Delta_{12} MMR_{t-1}$ | 4 | 1.098 | 5.356 |
| $\Delta_3 MMR_{t-1}$ | 3 | 0.276 | $MMR_{t-1}$ | 17 | 6.612 | 5.386 |
| $\Delta_3 MMR_{t-1}$ | 3 | 0.270 | $MMR_{t-2}$ | 30 | 6.612 | 5.448 |
| $\Delta_3 MMR_{t-1}$ | 2 | 0.276 | TIME | 7 | 105.4 | 5.450 |
| $\Delta_3 MMR_{t-1}$ | 2 | 0.276 | $INFGAP_{t-1}$ | 11 | 0.875 | 5.670 |
| $\Delta_3 MMR_{t-1}$ | 4 | 0.276 | $\Delta MMR_{t-1}$ | 30 | 0.146 | 5.724 |