Composition of the Government Spending and Behaviour of the Real Exchange Rate in a Small Open Economy

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Abstract

The effects of government spending on a small open economy (SOE) have attracted little attention in the New-Keynesian SOE literature. One exception is Monacelli and Perotti (2007). In this paper we extend their work in several dimensions. First, we include both asset holder and non-asset holder households in the model. Second, we assume that the total government spending consists of spending on consumption goods and transfers to households. Modelling the government spending in this way enables us to analyse the responses of macroeconomic variables to different types of government spending shocks. Our results show that the effect of different types of government spending on the real exchange rate is different. Although, a rise in the government consumption spending leads to a depreciation, a rise in transfers to households leads to an appreciation.

JEL Classification: E21, E62, E63.

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* All the views expressed in this paper belong to the author and do not represent those of the Central Bank of the Republic of Turkey, or its staff.
1. Introduction

There has recently been a renewed interest about the effects of variations in the government spending on private consumption. This topic has attracted the attention of researchers since theory and empirical evidence suggest opposite effects on private consumption. Although empirical studies indicate an increase in private consumption after a positive government spending shock, standard RBC and New-Keynesian models predict the opposite. Using the US data, Blanchard and Perotti (2002) and Fatas and Mihov (2001) report that government spending shocks are very persistent and lead to an increase in output. Both studies also report that the effect of a government spending on consumption is significant and positive. Similar results are reported for the UK, Germany and Australia by Perotti (2002). Using a different identification procedure, Mountford and Uhlig (2002) investigated the effects of balanced budget and deficit spending shocks and find that government spending shocks do not crowd out consumption but do crowd out residential and non-residential investment. The findings of Gali et al. (2007) support the results of Blanchard and Perotti.

While empirical studies report similar results about the effects of government spending shocks, the predictions of the standard theoretical models do not match the empirical results. In particular, standard RBC and New-Keynesian models fail to produce a positive consumption response and a positive correlation between consumption and hours worked after a government spending shock. These types of models consist of infinitely lived households that take decisions subject to their intertemporal budget constraint. Due to their optimisation, an increase in government spending reduces consumption because of a decrease in the present value of after-tax income.\(^1\) In other words, consumers are behaving in a Ricardian fashion. Fatas and Mihov (2001) argue that this negative wealth effect is a robust feature of the RBC models with different specifications, for example, with different financing options of government spending and different labour supply elasticities.

More recent literature propose different methods to improve the limited ability of the standard RBC and New-Keynesian models to replicate the effects of government spending shocks on macroeconomic variables. Linnemann (2006) shows that obtaining a positive consumption response after a government spending

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\(^1\) The transmission mechanisms of government spending shocks are discussed in more detail in Baxter and King (1993), Christiano and Eichenbaum (1992), and Fatas and Mihov (2001).
shock in a standard RBC model is possible by using a non-separable utility function. Gali et al. (2007), incorporate non-Ricardian consumer behaviour by including rule-of-thumb consumers into the model together with conventional Ricardian consumers and show that the coexistence of sticky prices and rule-of-thumb consumers is a necessary condition for a positive consumption response after a government spending increase.

Moreover, the effects of variations in government spending on the real exchange rate and net exports has attracted little attention in the theoretical literature. Monacelli and Perotti (2007) is one of the exceptions.\(^2\) First, they report empirical evidence from an SVAR model. They show that after a positive government spending shock the real exchange rate depreciates in the US, Australia, Canada and the UK. After two years, the real exchange rate appreciates only in Canada. The trade balance deteriorates in the UK, Canada and Australia. In the US, the effect is insignificant in the short run, however, it is small but significantly positive in the long run (after three years). Then, they show that although SVAR results indicate a depreciation, standard New-Keynesian models produce an appreciation of domestic currency after a positive government spending shock. They call this result “the real exchange puzzle”. They demonstrate that appreciation of domestic currency is the result of complete markets assumption and separable utility function. They also show that non-separable utility function can solve not only consumption puzzle but also the real exchange rate puzzle.

In this paper we extend the model in Monacelli and Perotti (2007) in several dimensions. First, we assume that the total government spending consists of both spending on consumption goods and transfers to households. The rationale of our assumption is the launch of the massive fiscal stimulus packages during the current financial crisis.\(^3\) These packages include various forms of fiscal policies: e.g. tax reductions, increase in government spending on consumption goods, infrastructure investments and increase in transfers to households. Modelling the government spending in this way enables us to analyse the responses of macroeconomic variables to different types of fiscal policy shocks. Secondly, we include non-asset holder households in the model. Therefore, we can analyse how two different household groups behave after different types of government spending shocks. Our

\(^2\) For others see Erceg et al. (2005) and Galstyan and Lane (2009).
\(^3\) The major economies that launched fiscal stimulus packages are the US, the UK, Canada, Germany, Japan, China and France.
results show that the effects of different types of government spending on the real exchange rate are different. Even though a rise in the government consumption spending leads to a depreciation, a rise in transfers to households leads to an appreciation.

The paper is organized as follows: In section 2, the model is introduced. Section 3 consists of equilibrium conditions. Section 4 outlines the calibration of the parameters. We discuss the puzzles and the proposed solutions in section 5. In section 6, we demonstrate the impulse-responses of macroeconomic variables to different government spending shocks. In addition, results of global sensitivity analysis are documented. Section 7 concludes the paper.

2. The Model

The model consists of a continuum of infinitely-lived households and has the feature of limited asset market participation in the sense that a fraction of households do not have access to the asset market. We call these households non-Ricardian households. The assumption regarding the existence of non-Ricardian consumers is motivated by Campbell and Mankiw (1989) and Mankiw (2000). Firms produce differentiated products and set prices on a staggered basis. The monetary authority sets interest rates according to an interest feedback rule. The fiscal authority raises income by imposing lump-sum taxes. Government spending consists of government spending on consumption goods and transfers to households. The rest of the world (ROW) consists of a continuum of small open economies as in Gali and Monacelli (2005). We also assume that the domestic economy (SOE) is relatively small compared to the ROW so that it cannot affect the ROW. On the other hand, shocks that originate in the ROW affect the SOE.

2.1. Households

We assume that a fraction of the households \((1 - \lambda)\) behave in a Ricardian fashion, smoothing their consumption by trading riskless one-period bonds and holding shares in monopolistically competitive firms. The remaining households \(\lambda\) do not have access to the asset market and consume their current after-tax income.

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4 Mishkin (1991) argues that institutional constraints are the main reason of limited asset market participation.
Asset Holders

The objective of the households which have access to the asset markets is to maximise their life time utility subject to their budget constraint. We use a non-separable utility function that belongs to the King-Plosser-Rebelo (1988) class:

$$\max E_{t,s=0}^{\infty} \beta^t \frac{C_{A,t+s}^{1-\sigma} (1 - L_{A,t+s})^{1+\phi}}{1-\sigma}$$  \hspace{1cm} (1)

where \( L_{A,t} \) denotes leisure and \( C_{A,t} \) is a composite consumption index of asset holders and defined by

$$C_{A,t} = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{A,H,t}^{\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{A,F,t}^{\frac{1}{\eta}} \right]^{\frac{1}{\eta}}$$

where \( 0 < \alpha < 1 \) indicates the share of imported goods in the consumption basket of households and \( \eta > 0 \) is the elasticity of substitution between domestic and foreign goods. \( C_{A,H,t} \) is an index of consumption goods produced in the SOE with the CES function

$$C_{A,H,t} \equiv \left[ \int_{0}^{1} C_{A,H,t}^{\frac{\gamma}{\gamma - 1}}(j) dj \right]^{\frac{\gamma}{\gamma - 1}}$$

where \( j \in [0,1] \) represents the differentiated goods produced in the economy. \( C_{A,F,t} \) is the CES index of consumption goods produced in the ROW

$$C_{A,F,t} \equiv \left[ \int_{0}^{1} C_{A,F,t}^{\frac{\gamma}{\gamma - 1}} di \right]^{\frac{\gamma}{\gamma - 1}}$$

where \( \gamma > 0 \) is the substitution between goods produced in different foreign economies. \( C_{A,i,t} \) is the index of the quantity of goods imported from country \( i \) and consumed in the SOE and can be written as a CES function

$$C_{A,i,t} \equiv \left[ \int_{0}^{1} C_{A,i,t}^{\frac{\epsilon}{\epsilon - 1}}(j) dj \right]^{\frac{\epsilon}{\epsilon - 1}}$$

Budget constraint of the Ricardian households is
\begin{equation}
E_t \left[ \Phi_{t,t+1}D_{A,t+1} \right] + \int_0^1 P_{H,t}(j)C_{A,H,t}(j) dj + \int_0^1 P_{i,t}(j)C_{A,i,t}(j) dj \, d\nu_t \\
+ P_t T_t = D_{A,t} + W_t N_{A,t} + P_t V_t \tag{2}
\end{equation}

$D_{A,t+1}$ is the nominal pay-off in period $t+1$ of the portfolio held at the end of period $t$, including the shares in firms. $\Phi_{t,t+1}$ is the stochastic discount factor. $R_t \equiv 1/E_t \left[ \Phi_{t,t+1} \right]$ is the gross return on a riskless one-period bond that pays off one unit of domestic currency in period $t+1$. $W_t$ is the nominal wage, $N_{A,t}$ is the hours worked by asset holders and $N_{A,t} = 1 - L_{A,t}$, $V_t$ is the government transfers to households. $T_t$ is the lump-sum taxes paid to the government.

The expenditure minimisation problem of Ricardian consumers implies the following demand functions:

\begin{align*}
C_{A,H,t}(j) &= \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{\epsilon} C_{A,H,t} \\
C_{A,i,t}(j) &= \left[ \frac{P_{i,t}(j)}{P_{i,t}} \right]^{\epsilon} C_{A,i,t}
\end{align*}

$i$ and $j \in [0,1]$, where $P_{H,t}$ is the price index of domestically produced goods and can be written as $P_{H,t} \equiv \left( \int_0^1 P_{H,t}(j) d\nu_t \right)$ and $P_{i,t} \equiv \left( \int_0^1 P_{i,t}(j) d\nu_t \right)$, is the price index of goods imported from country $i$ in domestic currency. Then, we can write \( \int_0^1 P_{H,t}(j)C_{A,H,t}(j) d\nu_t = P_{H,t} C_{A,H,t} \) and \( \int_0^1 P_{i,t}(j)C_{A,i,t}(j) d\nu_t = P_{i,t} C_{A,i,t} \).

Expenditure minimisation of Ricardian households for imported goods from country $i$ gives

\begin{equation}
C_{A,i,t} = \left[ \frac{P_{i,t}}{P_{F,i}} \right]^\gamma C_{A,F,t}
\end{equation}
where \( P_{F,t} \equiv \left( \int_{0}^{1} P_{i,t}^{1-\gamma} di \right)^{1/1-\gamma} \) is the price index of imported goods in domestic currency. Aggregate expenditure of Ricardian households on imported goods can be written as \( \int_{0}^{1} P_{i,t} C_{A,i,t} di = P_{F,t} C_{A,F,t} \).

It can be shown that aggregate demand functions of the Ricardian households for domestically produced and imported goods are

\[
C_{A,H,t} = (1 - \alpha) \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} C_{A,t}
\]

\[
C_{A,F,t} = \alpha \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} C_{A,t}
\]

where \( P_t = [(1 - \alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}] \) is the consumer price index.

The total expenditure of Ricardian household is \( P_t C_{A,t} = P_{H,t} C_{A,H,t} + P_{F,t} C_{A,F,t} \) and substituting into the budget constraint of Ricardian households yields:

\[
E_t[\Phi_{t,t+1} D_{A,t+1}] + P_t C_{A,t} + P_t T_t = D_{A,t} + (W_t, N_{A,t}) + P_t V_t
\]

The first order conditions of asset holders are

\[
R_t E_t[\Phi_{t,t+1}] = 1
\]  

(3)

\[
\Phi_{t,t+1} = \beta \left( \frac{C_{A,t}}{C_{A,t+1}} \right)^{\sigma} \left( \frac{N_{A,t+1}}{N_{A,t}} \right)^{1+\varphi} \frac{P_t}{P_{t+1}}
\]  

(4)

\[
\frac{W_t}{P_t} = \frac{(1 + \varphi) C_{A,t}}{(\sigma - 1) N_{A,t}}
\]  

(5)

After log-linearising the Euler equation of asset holders and using steady-state hours (5) yields

\[
c_{A,t} = c_{A,t+1} - \frac{1}{\sigma} (r_t - E_t \pi_{t+1}) - \frac{1 + \varphi}{\sigma} (n_{A,t+1} - n_{A,t})
\]  

(6)

where lowercase letters represent log deviations from the steady state. After log-linearisation, the labour supply of asset holders can be written as
where, $w_i$ is the real wage.

**Non-Asset Holders**

We assume that both types of households have the same preferences, therefore $\phi$ and $\sigma$ are the same for asset holders and non-asset holders.

Non-asset holders maximise their current utility

$$
\max \frac{C_{N,t}^{1-\sigma} (1 - L_{N,t})^{1+\phi}}{1 - \sigma}
$$

subject to the following budget constraint

$$
P_t C_{N,t} + P_t T_t = W_t N_{N,t} + P_t V_t
$$

where $C_{N,t}$ and $N_{N,t}$ are consumption and labour supply of non-Ricardian households respectively.

The aggregation procedure of budget constraints of the non-asset holders is very similar to the Ricardian household case; therefore it is not shown in detail.

The optimality condition for non-asset holders is

$$
\frac{W_t}{P_t} = \frac{(1 + \phi) C_{N,t}}{(\sigma - 1) N_{N,t}}
$$

Its log-linearised form can be written as

$$
n_{N,t} = c_{N,t} - w_i
$$

Log-linearised budget constraint of the non-asset holders is

$$
c_{N,t} = \frac{1}{(1 - G_{CY})(w_i + n_{N,t})} - \frac{T_y}{(1 - G_{CY})} t_i + \frac{V_y}{(1 - G_{CY})} v_t
$$

where $T_y$ is lump-sum tax revenue divided by output, $T_y = \frac{T}{Y}$, and $G_{CY}$ is the share of government consumption goods spending in output ($G_{CY} = \frac{G^C}{Y}$) at the steady state.
Aggregate Consumption and Labour Supply

In order to simplify the derivations, we assume that hours worked in steady state are the same for both types of household, \( N_A = N_N = N \). Then, labour market clearance implies the following aggregate relationship for the labour supply:

\[ n_t = \lambda n_{N,t} + (1 - \lambda) n_{A,t} \]

In addition, homogeneity of preferences ensures that marginal rates of substitution of both types of household will be equalised in steady state. We also eliminate steady state profit by setting \( F_t = \Theta \). As a result, steady state consumptions are \( C_A = C_N = C \). Hence, aggregate consumption can be written as

\[ c_t = \lambda c_{N,t} + (1 - \lambda) c_{A,t} \]

2.2. Inflation and the Real Exchange Rate

We assume that the law of one price holds for each good. The bilateral real exchange rate between SOE and country \( i \) is defined as \( q_{t,i} = \frac{\varepsilon_{i,t} P^i_t}{P_t} \), where \( \varepsilon_{i,t} \) is the nominal exchange rate (domestic currency price of country \( i \)'s currency) and \( P^i_t \) is the aggregate price index of country \( i \)'s consumption goods. After aggregation and log-linearisation the real exchange rate can be written as

\[ q_t = p_{F,t} - p_t \]

where \( p_{F,t} \) is the price of foreign goods in domestic currency, \( (p_{F,t} = \varepsilon_{i} P^F_{t,i}) \).

Then, using the log-linearised formula of CPI around a symmetric steady state the domestic price level and the real exchange rate can be linked through

\[ p_t = p_{H,t} + \frac{\alpha}{1 - \alpha} q_t \]  \hspace{1cm} (12)

2.3. Firms

Intermediate Good Firms

Intermediate good firms are monopolistically competitive and produce a differentiated good. Output linearly depends on labour with the following production function

\[ Y_t(i) = N_t(i) - F(i) \]  \hspace{1cm} (13)
where \( i \in 0,1 \) and \( F(i) \) is a firm specific fixed cost. Firms produce intermediate goods as long as \( N_i(i) > F(i) \), otherwise \( Y_i(i) = 0 \). Existence of the firm-specific fixed cost ensures the increasing returns to scale consistent with the Rotemberg and Woodford (1995). It is also possible to restrict the profits of firms to zero at steady state by choosing the firm-specific fixed cost appropriately.

Log-linearised aggregate output can be written as
\[
y_i = n_i(1 + F_y)
\]
where \( F_y \) is the ratio of fixed cost to output ratio at the steady state.

Cost minimisation of the firms lead to the following nominal marginal cost function for the firms
\[
MC^\pi_i = W_i
\]
(14)

The log-linearised real marginal cost of a firm can be derived using equations (12) and (14) as
\[
m_c = w_i + \frac{\alpha}{1-\alpha} q_i
\]
(15)

**Final Goods Firms**

The representative firm, which produces the final output, is a competitive firm. This firm produces the final good using the intermediate goods produced by monopolistically competitive firms. The aggregation technology of the final good firm is in the CES form and has the property of constant elasticity of substitution, \( \varepsilon \).

\[
Y_i = \left( \frac{1}{0}i Y_i(i)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon-1}{\varepsilon}}
\]
(16)

where \( Y_i(i) \) is the quantity of the differentiated good \( i \) used in the production of final good. The demand function of the final goods producer for each intermediate output is
\[
Y_i(i) = \left( \frac{P_i(i)}{P_{Hi,i}} \right)^{-\varepsilon} Y_l
\]
(17)
**Price Setting**

We assume that intermediate good firms set prices according to a Calvo (1983) framework in which only a randomly selected fraction, \(1 - \theta\), of firms can adjust their prices optimally. Thus, \(\theta\) is the probability that firm \(i\) does not change its price in period \(t\). Then firm \(i\) sets price \(P_t(i)\) by solving the following problem

\[
\max E_i \sum_{s=0}^{\infty} \theta^s \Phi_{t+s} \left[ P_t(i) Y_{t,t+s}(i) - W_{t+s} Y_{t,t+s}(i) \right] 
\]

subject to the demand function (17). The first order condition for this problem is

\[
E_i \sum_{s=0}^{\infty} \theta^s \Phi_{t+s} \left[ P_t(i) - \frac{\varepsilon}{1-\varepsilon} W_{t+s} \right] = 0 
\]

Firms that set a new price \(P_t(i)\) at time \(t\), will choose the same price and output at equilibrium.

Aggregating over \(i\) and taking the log linear approximation of equation (19) gives us the price setting equation

\[
\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \mu mc \wedge
\]

where \(\mu = (1 - \theta)(1 - \theta\beta)/\theta\) and \(mc\) is deviation of the marginal cost from the constant steady state marginal cost.

**2.4. Monetary Policy**

We assume that monetary policy is conducted according to the following simple Taylor type monetary policy rule

\[
r_t = \phi_r \pi_t 
\]

where \(\pi_t = \log(P_t/P_{t-1})\) is the CPI inflation between period \(t\) and \(t+1\). The response of the monetary authority to inflation is governed by \(\phi_r\).

**2.5. Fiscal Policy**

The fiscal authority collects lump-sum taxes, \(T_t\). We divide the total government spending, \(G_t\), into two categories; government spending on consumption goods, \(G^C_t\), and government transfers to households, \(V_t\). Designing government spending in this way allows us to investigate the transmission mechanisms of different
government spending shocks. We assume that the government of the SOE and that of the ROW are home-biased and hence, only consume domestic goods. Total government spending is 

\[ G_t = G_t^c + V_t \]

after log-linearisation it can be written as 

\[ g_t = C^g g_t^c + V^g v_t \]

where, \( C^g \) is share of consumption good spending and \( V^g \) is share of transfers to household in total government spending at steady state. Log linearised \( g_t^c \) and \( V_t \) are defined as 

\[ g_t^c = \left( G_t^c - G_t^c^* \right) / G_t^c^* \quad \text{and} \quad v_t = \left( V_t^c - V_t^* \right) / V_t^* \]

and both follow AR(1) processes 

\[ g_t^c = \rho_g g_{t-1}^c + \xi_t^c \]

\[ v_t = \rho_v v_{t-1} + \xi_t^v \]

where \( \xi_t^c \) and \( \xi_t^v \) are i.i.d. government consumption goods spending and government transfers households shocks with variances \( \sigma^2_{\xi_t^c} \) and \( \sigma^2_{\xi_t^v} \). 

The government’s budget constraint is 

\[ G_t = T_t \quad (22) \]

and after log-linearisation 

\[ G_t g_t = T_t t_t \]

where \( G_t \) is the total government spending to output ratio at steady state. 

2.6. International Risk Sharing 

Households, who have access to the asset markets in country \( i \) are able to invest in the SOE. Therefore, equation (4) must hold for asset holders in country \( i \) as well: 

\[ \Phi_{t,t+1} = \Phi_{t,t+1} = \beta \begin{bmatrix} C_{A,t}^i & N_{A,t+1}^i \end{bmatrix} \begin{bmatrix} \sigma^i \end{bmatrix}^{1+\phi} \begin{bmatrix} p_t^i \end{bmatrix} \begin{bmatrix} \varepsilon_{e,i,t} \end{bmatrix} \begin{bmatrix} e_{i,t+1}^j \end{bmatrix} \quad (23) \]

\(^3\) Schmitt-Grohe and Uribe (2005) use a similar structure to study Ramsey optimal fiscal and monetary policies in a closed economy model.
Note that price of the security and security’s one unit payoff are converted to country i’s currency. After rearranging (23), we get

$$C_{A,i} = C_{A,i}^{\gamma} \left[ \frac{N_{A,i}}{N_{A,i+1}^{\gamma}} \right]^{(1+\phi)\sigma} \left( \frac{N_{A,i+1}^{\gamma}}{N_{A,i+1}^{\gamma}} \right)^{\frac{1}{\sigma}} C_{A,i+1}^{\gamma} \frac{1}{q_{i+1}^{\sigma}}$$

(24)

Let

$$\left[ \frac{N_{A,i}^{\gamma}}{N_{A,i+1}^{\gamma}} \right]^{\frac{1}{\sigma}} C_{A,i+1}^{\gamma} \frac{1}{q_{i+1}^{\sigma}} = \vartheta_i$$

where \( \vartheta_i \) is a constant and generally depends on the initial relative asset positions. Then, equation (24) can be written as

$$C_{A,i} = C_{A,i}^{\gamma} \left[ \frac{N_{A,i}}{N_{A,i}^{\gamma}} \right]^{\frac{1+\phi}{\sigma}} \vartheta_i q_{i+1}^{\sigma}$$

(25)

Assumption of net initial asset position being zero for every pair of countries leads to \( \vartheta_i = 1 \). We take the log of equation (25) and then integrate over \( i \) to get the risk sharing between the asset holders:

$$c_{A,i} = c_{A,i}^{*} + \frac{1+\phi}{\sigma} (n_{A,i} - n_{A,i}^{*}) + \frac{1}{\sigma} q_i$$

(26)

where \( c_{A,i}^{*} \) and \( n_{A,i}^{*} \) are asset holders’ consumption and labour supply in the ROW, respectively.

3. Equilibrium Conditions

3.1. Goods Market Equilibrium

We assume that foreign and domestic governments are home biased but households consume both domestic and foreign goods. Then, the goods market equilibrium requires

$$Y_i(j) = C_{H,i}^{l}(j) + \int_{0}^{1} C_{H,i}^{l}(j) di + G_i^{c}(j)$$

(27)

where \( j \) is a good produced in the domestic country and \( C_{H,i}^{l}(j) \) is the domestic demand for good \( j \), \( C_{H,i}^{l}(j) \) is country i’s demand for good \( j \), \( G_i^{c}(j) \) is the
domestic government’s demand for good \( j \). As explained in detail in Appendix B1, an optimal allocation of expenditures between domestic and foreign goods and the assumption that \( \gamma = \eta \) implies the following aggregate demand equation

\[
y_t = (1 - G_{CY})c_t + G_{CY}g^c_t + (1 - G_{CY}) \left[ \frac{\gamma \alpha}{1 - \alpha} + \alpha(\gamma - \frac{1}{\sigma}) \right] q_t.
\] (28)

3.2. Net Exports

Following Gali and Monacelli (2005) we define net exports as

\[
x_t = (Y_t - G_t^c - \frac{P_t}{P_{H,t}} C_t) \frac{1}{Y}.
\] (29)

Log-linearising (29) gives us

\[
x_t = y_t - (1 - G_{CY})c_t - G_{CY}g^c_t - (p_t - p_{H,t})
\]

Using (12), substitute \((p_t - p_{H,t})\) with \(\frac{\alpha}{1 - \alpha} q_t\) to obtain

\[
x_t = y_t - (1 - G_{CY})c_t - G_{CY}g^c_t - (1 - G_{CY}) \left[ \frac{\alpha}{1 - \alpha} \right] q_t.
\] (30)

Equation (30) implies that the net exports of each country is zero at steady state.

4. Baseline Calibration

Time is measured in quarters. Consistent with the extant literature, we set \( \beta = 0.99 \), implying a riskless annual return of approximately 4% in steady state. The inverse of the elasticity of intertemporal substitution, \( \sigma \), is taken as 3. The inverse of the elasticity of labour supply \( \phi \) is determined according to (33). We set the openness parameter \( \alpha \) to 0.4. Febris and Winer (2007) analyse fiscal data of Canada in detail. We follow them while calibrating the fiscal side of the model. The government’s share in the economy is 36.2 percent. Share of government transfers to households in total output is 11.2 percent. Following most of the literature the steady state debt to output ratio, \( B/Y \), is taken as zero. The gross markup is set as 1.2. Following Botman et al. (2006) we set the share of non-Ricardian households in the economy as 20 percent. AR(1) parameters of the shocks are taken from Schmitt-Grohe and Uribe (2005). Baseline parameter values are summarised below.
\[ \beta = 0.99 \text{ Discount factor} \]
\[ \sigma = 3 \text{ Inverse of the intertemporal elasticity of substitution} \]
\[ \lambda = 0.25 \text{ Share of non-Ricardian households} \]
\[ \theta = 0.75 \text{ Calvo parameter} \]
\[ \phi = 1.5 \text{ Coefficient of inflation in the monetary policy rule} \]
\[ \eta = \gamma = 0.5 \text{ Elasticity of substitution between domestic and foreign goods} \]
\[ \alpha = 0.4 \text{ Degree of openness} \]
\[ G_y = 0.4 \text{ Share of government spending in output} \]
\[ V_c = 0.112 \text{ Share of transfers to households in output} \]
\[ C^x = 0.657 \text{ Share of consumption good spending in total government spending} \]
\[ V^t = 0.343 \text{ Share of transfers to households in government spending} \]
\[ G_{c^t} = 0.237 \text{ Share of government’s consumption good spending in total output} \]
\[ \rho_c = 0.87 \text{ AR(1) coefficient of the government consumption good spending} \]
\[ \rho_i = 0.78 \text{ AR(1) coefficient of the government transfers} \]

5. Reconciliation of the Theory with the Empirical Evidence

5.1. Solving the Consumption Puzzle

In standard RBC and New-Keynesian models, the log-linearised Euler equation of an intertemporal optimising household is

\[ c_{A,t} = E_t(c_{A,t+1}) - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) \]

In this setting, when the government increases its spending, the present value of the tax burden increases. A resulting negative wealth effect forces Ricardian consumers to reduce their consumption. Persistence of government spending is one of the factors that determines the present discounted value of taxes. Lower persistence implies a shorter period of budget deficits, and lower negative wealth effects due to lower future tax burdens for asset holders. An additional transmission channel which affects the consumption decision of Ricardian agents is the response of monetary policy to the inflationary effects of government spending shocks. A stronger response of interest rates to inflation implies a higher substitution of current consumption for future consumption.
Two different routes are taken in the literature to produce a positive consumption response after a government spending shock. The first approach enables a shift in labour demand after the government spending shock via counter-cyclical mark-ups or non-Ricardian consumers. In this type of model, wages rise if the increase in labour demand is higher than the increase in labour supply and higher wages boost consumption. Devereux et al. (1996) and Ravn et al. (2006a) use a model with counter-cyclical mark-ups and show that wages increase if labour demand increases sufficiently; hence households substitute leisure for consumption, as a result consumption increases.

Gali et al. (2007) introduce rule-of-thumb (non-Ricardian) consumers with nominal rigidities in order to generate a positive consumption response after the government spending shock. Consumption of non-Ricardian household depends on real wages, hours worked and taxes. The real wage is determined by the dynamic interaction of labour supply and demand in the labour market. The labour demand of firms depends on the degree of price stickiness in the economy. When the demand for goods increases after a fiscal spending shock, $(1 - \theta)$ percent of firms adjust their prices. On the other hand, $\theta$ percent of the firms are not able to reset their prices. They respond to the increased demand for their product by increasing output which raises demand for labour. Note that in such a situation a higher degree of price stickiness implies higher labour demand. The labour supply of non-Ricardian households is determined by their disposable income. If government spending is partly financed by higher taxes, the disposable income of non-Ricardian consumers declines, hence they will want to work more. If the deficit is completely financed by issuing debt, then the labour supply of non-asset holders does not change and their consumption is determined solely by the change in the real wage. If the share of non-Ricardian consumers is sufficiently high in the economy, then it is possible to obtain an increase in consumption after the government spending increase.

A second approach is taken by Basu and Kimball (2002) and Linnemann (2005) by introducing non-separability in preferences between leisure and consumption. The advantage of a non-separable utility function is that it enables us to obtain the positive relationship between current hours worked and consumption found in the data. Euler equation of asset holders can be written in our model as

$$E_t(\Delta c_{A,t+1}) = \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) + \frac{1+\phi}{\sigma}E_t(\Delta n_{A,t+1})$$

(31)
In this setting, expected consumption growth not only depends on the expected real interest rate but also the expected change in the labour supply. Suppose our economy consists of only Ricardian consumers \((\lambda = 0)\). In this case, when the government spending increases, due to negative wealth effects, we expect Ricardian households to increase their labour supply. Equation (31) ensures that the increase in hours of work increases consumption given the expected real interest rate.

5.2. Solving the Real Exchange Rate Puzzle

Structural VAR models show that government spending shocks lead to a depreciation of the domestic currency. However, standard RBC and New-Keynesian models predict the opposite. Specifically, the appreciation of domestic currency after a government spending shock is a robust feature of the theoretical models which assume complete markets. The reason is that in these kinds of models, the real exchange rate is determined by an international risk sharing condition. In a standard open economy New-Keynesian model international risk sharing implies that

\[ q_t = \sigma(c_t - y^*_t) \]

As \( y^*_t = 0 \) in the absence of foreign shocks, in the case of a domestic government spending shock the real exchange rate follows domestic consumption proportionally. Since domestic consumption declines after the government spending shock due to negative wealth effects the real exchange rate appreciates in these models. Inclusion of non-Ricardian households into the model doesn't solve the puzzle since the exchange rate is determined according to the consumption behaviour of Ricardian households. Monacelli and Perotti (2007) report that this result is robust in the presence of traded and non-traded goods, local currency pricing and pricing to market specifications.

Monacelli and Perotti (2007) show that non-separability of consumption and leisure ensures the depreciation of the real exchange rate after a positive government spending shock. Typical log-linearised international risk sharing equation in this type of model is reported in equation (26). In the domestic government spending case \( c^*_{A,t} = 0 \) and \( n^*_{A,t} = 0 \). Then, in our model, the risk sharing equation (26) reduces to

\[ q_t = \sigma c^*_{A,t} - (1 + \varphi)n^*_{A,t} \]

(32)
Equation (32) shows that the real exchange rate depends on consumption and the labour supply of asset holders. Negative wealth effects caused by an increase in the government spending forces asset holders to work more, increasing the hours worked. If the model produces positive consumption after the government spending shock then both \( c_{A,t} \) and \( n_{A,t} \) will be positive after the shock hits the economy. Then, the path of the real exchange rate is determined by the coefficients \( \sigma \) and \( \varphi \). As shown in Appendix B, value of \( \varphi \) is not independent from other parameter values and the steady state condition implies the following relation for \( \varphi \):

\[
\varphi = \frac{(\sigma - 1)}{(\sigma - G_{cy})} - 1
\]

(33)

where \( \varphi > 0 \).

6. The Transmission Mechanism of Different Government Spending Shocks

We divide the total government spending into the government spending on consumption goods and the government transfers to households.\(^6\) Transmission mechanism of these two fiscal policy tools are different especially if non-Ricardian households exist in an economy. An increase in government consumption spending directly raises the aggregate demand through goods market equilibrium. On the other hand, a rise in transfers to households do not have a direct affect on the aggregate demand. But transfers affect the labour supply and consumption decisions of non-Ricardian households directly. Therefore, existence of non-Ricardian households makes the transmission channels of government spending shocks even more complicated.

6.1. Impulse-Response Analysis

We report the effects of an increase in government transfers to households and government spending on consumption goods in Figure 1 and Figure 2 respectively. We calibrate the standard deviation of each shock so that the increase in total government spending is 1 percent for each shock.

Case I: A Rise in the Government's Consumption Good Spending

Directions of the responses of total consumption, output, and employment are consistent with the empirical findings. Increase in output can be attributed to the

\(^6\) For the government consumption and investment spending cases in a small open economy model see Galstyan and Lane (2009).
sharp rise in labour supply of non-Ricardian households. Substituting (11) into (10) gives:

\[ n_{N,t} = -w^*_t + t^*_t \frac{V}{T^*_t} \]

(34)

Accordingly, the labour supply of non-Ricardian households declines with higher real wages and government transfers (assuming \( T^*_t / V > 0 \)) but increases with higher lump-sum taxes. In the absence of a shock to government transfers to households labour supply of non-Ricardian households is determined by the real wage and lump-sum taxes. In the model, an increase in government’s consumption goods spending raises lump-sum taxes and reduces wages. Therefore, both variables push labour supply of non-Ricardian households, hence output, up. The real exchange rate depreciates and net exports decline, which is consistent with Monacelli and Perotti (2007). Although we do not report the results, we note that response of net exports is quite sensitive to the elasticity of substitution between domestic and imported goods, \( \eta \). In the model, \( \eta < 0.8 \) ensures a deterioration in net exports. Hooper et al. (2000) report that \( \eta \) varies between 0.1 and 2 in G-7 countries. We set \( \eta = 0.5 \) that is consistent with the empirical evidence.

Case II: A Rise in Government Transfers to Households

Compared with the first case, responses of output and total consumption are still positive but smaller in magnitude. Effects of rising lump-sum taxes dominates the increase in real wages, hence non-Ricardian households rise their labour supply. On the other hand, Ricardian households reduce their labour supply. Since labour supply responses of different household groups are in opposite way, aggregate labour supply increases very little. As a result, increase in output as well as total consumption remains limited compared to the first case. In response to an increase in transfers to households, the real exchange rate appreciates and net exports improves.\(^7\)

Although total consumption rises after a government transfer, consumption of Ricardian households decline. Cross country evidence about the responses of different types of households after a government transfers shock is limited. Johnson et al. (2006) report that the US consumers increase their consumption spending

\(^7\) Galstyan and Lane (2009) find that government consumption good spending and investment spending shocks lead to different outcomes for the real exchange rate.
after the 2001 US Federal tax rebate. In addition, they report that response of households holding relatively less assets is higher than the other households. We believe that further evidence is needed about the response of Ricardian households to government transfers shocks.

**Global Sensitivity Analysis**

We carry out a global sensitivity analysis to understand which parameters are more important for the stability of the equilibrium and report our results in Figure 2.3. The shaded area shows the combinations of parameter values that lead to unstable equilibrium in the model. Our results show that calibrations of $\gamma$ and $\phi_{\pi}$ are crucial for the stability of the model. Holding other parameters constant, coefficient of monetary policy rule, $\phi_{\pi}$, must be greater or equal to one and elasticity of substitution between domestic and imported goods, $\eta$, must be between zero and two.

**7. Conclusion**

In the last decade, many researchers have tried to reconcile the empirical evidence on the effect of government spending on private consumption with the theoretical findings. However, there are limited efforts in the recent literature to explain the effects of government spending shocks on a small open economy. Monacelli and Perotti (2007) is an exception. In this paper, we extend their work in several dimensions. First, we include both Ricardian and non-Ricardian households into the model. Second, we assume that a government can increase its spending by either raising its demand for consumption goods or raising transfers to households.

Our interest is to analyse how qualitative comovements of the real exchange rate, trade balance and private consumption change with the inclusion of non-Ricardian households and different government spending shocks. Therefore, our purpose is not to fit the model results with the data but to compare the signs of the responses with empirical findings. In the baseline calibration, signs of the responses of output, total consumption and net exports are consistent with the data for both types of government spending shocks. The real exchange rate depreciates after a government consumption spending shock and appreciates after a transfer to households shock.

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8 We have used the global sensitivity analysis toolbox developed by Marco Ratto. See Ratto (2008) for details.
In other words, response of the real exchange rate depends on the nature of the government spending shocks.

Global sensitivity analysis results show that proper calibration of the parameters representing elasticity of substitution between domestic and imported goods and responsiveness of the interest rates to inflation are important for the stability of the model.
Appendix A. Goods Market Equilibrium

We assume that domestic and foreign governments are home biased. Then, the goods market equilibrium requires

\[ Y_i(j) = C_{H,i}(j) + \int_0^1 C_{H,j}(j) \, dj + G_i^c(j) \]  

where, \[ C_{H,i}(j) = \left( \frac{P_{H,i}(j)}{P_{H,t}} \right)^{-\alpha} \left( \frac{P_{H,j}}{P_{H,t}} \right)^{-\gamma} C_i \] and

\[ G_i^c(j) = \left( \frac{P_{H,i}(j)}{P_{H,t}} \right)^{-\gamma} G_i^c. \]

Assuming symmetric preferences across countries, demand function of consumer of country \( i \) for good \( j \) can be written as

\[ C_{H,i}(j) = \alpha \left( \frac{P_{H,i}(j)}{P_{H,t}} \right)^{-\alpha} \left( \frac{P_{H,j}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,j}}{P_{F,t}} \right)^{-\gamma} C_i \]

by using \[ C_{H,i}(j) = \left( \frac{P_{H,i}(j)}{P_{H,t}} \right)^{-\alpha} \left( \frac{P_{H,j}}{P_{H,t}} \right)^{-\gamma} \left( \frac{P_{F,j}}{P_{F,t}} \right)^{-\gamma} C_i \] and \[ C_{F,j} = \alpha \left( \frac{P_{F,j}}{P_{F,t}} \right)^{-\gamma} C_i \]

After doing the necessary substitutions Equation (35) can be written as

\[ Y_i(j) = \left[ \frac{P_{H,i}(j)}{P_{H,t}} \right]^{-\gamma} \left( 1 - \alpha \right) C_i + \alpha \int_0^1 \left[ \frac{P_{H,j}}{P_{H,t}} \right]^{-\gamma} \left[ \frac{P_{F,j}}{P_{F,t}} \right]^{-\gamma} C_i \, dj + G_i^c \]  

Integrating over \( j \) using \[ Y_i = \left[ \int_0^{e^{-1}} Y_i(j) \, dj \right]^{e^{-1}} \] yields

\[ Y_i = \left[ \frac{P_{H,i}}{P_{F,t}} \right]^{-\gamma} \left( 1 - \alpha \right) C_i + \alpha \int_0^{1} Q_{H,i}^e C_i \, dj + G_i^c \]

taking the first order approximation of the equation above around the symmetric steady state and using \( p_i - p_{H,i} = \frac{\alpha}{1 - \alpha} \), \( \bar{Y}_i = \bar{C}^c + \bar{G}^c \) and \( \bar{C}_Y = \bar{C}^c \), we can write

\[ y_i = (1 - G_i^c) c_i + G_{cY} g_i + (1 - G^c_Y) \left[ \frac{\gamma \alpha}{1 - \alpha} + \alpha (\gamma - \frac{1}{\sigma}) \right] q_i \]

where \( G_{cY} \) is share of government's consumption good spending in output at steady state.
Appendix B. Steady State

From first order condition of the firm's problem, (19), for the steady state we can write

\[
\frac{W}{P} = \frac{\varepsilon - 1}{\varepsilon} \tag{38}
\]

Let \( \Theta = \frac{1}{\varepsilon - 1} \), and using \( Y = N - F \) in the steady state (38) can be written as

\[
\frac{W}{P} = \frac{Y + F}{N(1 + \Theta)} = \frac{Y + F}{N(1 + \Theta)} \tag{39}
\]

Steady state profit, \( \hat{\sigma} \), implies

\[
\hat{\sigma} = Y - \frac{WN}{P}
\]

Profit to output ratio is

\[
\hat{\sigma}_y = 1 - \frac{WN}{PY}
\]

or using the (39)

\[
\hat{\sigma}_y = 1 - \frac{1 + FY}{(1 + \Theta)} = \frac{(\Theta - FY)}{1 + \Theta} \tag{40}
\]

Setting \( \Theta = FY \) ensures that profit to output ratio is zero. Intertemporal consumption and leisure condition implies that

\[
\frac{W}{P} = \frac{(1 + \varphi)}{(1 - \tau)(\sigma - 1)} \frac{C}{N} \tag{41}
\]

dividing both sides to \( Y \)

\[
\frac{WN}{PY} = \frac{(1 + \varphi)}{(1 - \tau)(\sigma - 1)} \frac{C}{Y} \tag{42}
\]

At steady state, aggregate demand will be \( Y = C + G \). Dividing both sides by \( Y \) we get \( 1 = \frac{C}{Y} + \frac{G}{Y} \). Let \( G_y = \frac{G}{Y} \), then \( \frac{C}{Y} = (1 - G_y) \). Assuming, \( \Theta = FY \) in the steady state leads to \( \frac{WN}{PY} = 1 \). After substitutions, we can write

\[
1 = \frac{(1 + \varphi)}{(1 - \tau)(\sigma - 1)}(1 - G_y) \tag{43}
\]

using (43) \( \Phi \) will be equal to

\[
\varphi = \frac{(1 - \tau)(\sigma - 1)}{(1 - G_y)} - 1
\]

Steady state lump-sum tax can be driven using (8) and (41) as follows
\[ C_N = (1 - \tau) \frac{W_N}{P} - T \]

Note that at steady state, \( C_N = C_A = C \) and \( N_N = N_A = N \). Therefore, after substitutions, we can write the above equation as a share of output

\[ \frac{C}{Y} = (1 - \tau) \frac{W_N}{PY} - T_Y \] (42)

where \( T_Y = \frac{T}{Y} \).

\[ (1 - G_Y) = (1 - \tau) - T_Y \] (43)

Equation (43) implies that lump-sum taxes to output ratio in the steady state is determined according to the following equation:

\[ T_Y = G_Y - \tau \]
References


Figure 1. Responses to a Positive Government Consumption Good Spending Shock

- **output**
- **consumption**
- **total government spending**
- **real exchange rate**
- **net export**
- **expected real interest rate**
- **consumption-Ricardian HH**
- **consumption-non-Ricardian HH**
- **expected real interest rate**
- **labour-Ricardian HH**
- **labour-non-Ricardian HH**
- **real wage**
Figure 2. Responses to a Positive Government Transfers to Households Shock

- Output
- Consumption
- Total Government Spending
- Real Exchange Rate
- Net Export
- Expected Real Interest Rate
- Consumption - Ricardian HH
- Consumption - Non-Ricardian HH
- Labour - Ricardian HH
- Labour - Non-Ricardian HH

Figure 3. Indeterminacy Region $\theta_{PH}$ vs $\gamma$